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Y Do I Have to Care About an X? Struggling 6th Grade Students Take on Algebra

In the End...

In February, there were some serious battles being waged within the room! On a given day, my students complete a problem on their own and then check with the people around them to compare answers and discuss strategies. Little informal groups have formed during the year. On this cold snowy morning two alliances had been formed regarding the solution to a problem. Group A had typically been the group that usually had the correct answer and the other groups would refer to them to see if they were right. Not this day. Group B was absolutely convinced that they had the correct answer, and moreover, could prove they had the correct answer. The problem of the day was “Find 3 consecutive even numbers whose sum is double the third number, increased by 8.” Watching the two groups grapple with this problem put me in a ‘happy, sunny, no snow, no sleet, no cold, no solid precipitation of any kind’ state-of-mind. Not only was Group B victorious, they were able to convince Group A to their thinking through explanation.

This particular example is worth noting for several reasons. For one thing, the math involved in this problem is quite sophisticated. This is a classic abstract algebra problem, the kind of problem that makes all math-phobes quake. Yet, here were my sixth graders tackling it. But it was more than just the success that the students had with this problem. Quite simply, they were not afraid. They were not afraid of the words. They were not afraid of the computation involved in the problem. They were not afraid to defend their work. They were not afraid to try. Maybe on my quest to prove one thing, I discovered another. Maybe the magic of model-drawing and Hands-On equations isn’t the academic success that can be achieved. Maybe the magic is that it makes kids brave.

And maybe I should have had more faith that this day would come. Back in September, I wouldn’t have believed it to be possible.

James Otis World Language Academy: A Neighborhood School

The enrollment of this West Town area urban public school is currently 640 students in grades prekindergarten to eighth. There is a general education program, a bilingual program, and a special education program. The special education program includes a department for students with autism, a department for students with visual impairments (including blind and low-vision), and a cross-categorical department for students with learning disabilities and/or behavior problems. The majority of the students with IEPs are included in the general education

program for a significant part of the day. The majority of the students are classified as low-income, with 95% of the students qualifying for free lunch.

At one hundred thirty-eight years of age, this school has resided in what has been called a “port-of-entry” neighborhood. As immigrants have arrived in Chicago, many of the populations have settled in this community before becoming more upwardly mobile financially. In its history this school has transitioned from a predominantly Swedish population, to a predominantly Polish population, to a predominantly Italian population, and most recently, to a predominantly Hispanic population, specifically Mexican with some Puerto Rican and other Hispanics (e.g. Guatemalan, Ecuadorean). Otis has a small percentage of African-American students. Gentrification in the last ten years has forced many families of this community to move to other areas of the city that are more affordable. However, many of these families continue to attend this school by either driving their children or by using the CTA. Some students travel by CTA almost an hour a day to get to school.

This is a neighborhood school that does not “cherry pick” its students. The most recent available data for ISAT is from 2012. The school’s composite was 79.8% on the ISAT, with the CPS average being 76.4%. For the breakdown, the reading was 77.3% (CPS average – 73.4%), the mathematics was 84% (CPS average – 80.5%) and the science was 75% (CPS average – 72.9%). The school has been on and off probation over the years, but has consistently demonstrated growth on ISAT.

Two years ago the school absorbed the population of a school that was closed down due to poor test scores and low enrollment. This year, it absorbed another school that was closed due to low test scores on the ISAT. One hundred seventy-five students enrolled from this closed school. This has significantly increased its special education department, adding three new cross-category teachers to the department for learning disabled and/or behavior problems. The bilingual department has also grown due to the additional enrollment from the closed school. It went from three bilingual teachers to six bilingual teachers to accommodate the transferred students.

This school has done extremely well on the *5Essentials CPS My Voice, My School Survey*. Students, parents, and teachers participate yearly in this survey. The five categories are: a) Ambitious Instruction, b) Effective Leaders, c) Collaborative Teachers, d) Involved Families, and e) Supportive Environment. In the categories of Ambitious Instruction, Collaborative Teachers, Involved Families, and Supportive Environment, this school received Very Strong, the highest possible ranking. Effective Leaders received Strong, the next highest ranking, missing Very Strong by a few points. The composite for the school fell on the high end of the highest ranking, Well-Organized. The CPS composite fell on the low end of Organized, the next highest ranking.

There is very little transiency within the staff of the school. Excluding the new hires this school year, most of the staff has been working at this school for more than ten years. One teacher is in her 46th year of teaching at this school! Among this committed and professional teaching community, many have at least one Masters Degree. Three teachers are Nationally Board Certified and one teacher just finished his PhD. The teachers often have “kids of kids” in their classrooms and jokingly refer to themselves as the students’ grandparents. The aides assigned to the Special Education Department have also been working at this school for many years and are committed to the well-being of all of the students. This is a solid community that has enjoyed strong leadership. In the last twenty-two years, there have been only two principals. The current principal has been leading this community for six years. He has worked hard to continue the long tradition of high expectations for students and staff alike. He supports his staff and encourages them to try new methodologies without throwing out tried-and-true programs.

Keeping class size low at all grade levels has been a priority for the administration. Discretionary funds have been used to “buy” classroom teaching positions. Most rooms have an average of twenty-four students. Primary rooms are kept in the high teens whenever possible. There are occasional exceptions some years. When this occurs, as many auxiliary staff as possible are utilized to help out with the over-crowded rooms, especially during language arts instruction.

I am one of three sixth-grade classroom teachers this year. All classrooms have heterogeneous groupings. I have the largest group of students, twenty-three. There are fifteen girls and eight boys. Two students leave for special education services during language arts instruction. One of those students also has special education minutes in mathematics instruction. According to the ISAT, most of the students are below grade level in reading and mathematics. On the Fountas and Pinnell, only four students tested at grade level at the beginning of this year. Eight students tested in the fifth grade range, and the rest of the students are reading at about the beginning of fourth grade. Only one student tested Exceeds the Standard on ISAT mathematics. Most are listed in the Below category, with three students in Academic Warning. This group has been historically low, especially in mathematics, on the state test.

As a self-contained classroom teacher, I teach the same group of students in the subjects of language arts, mathematics, science, and social studies. This being my twenty-third year of teaching the same grade level, I have created and implemented many cross-subject area units. With supplies purchased using the numerous grants I have won and supplemental materials purchased by the school, I am able to integrate the subject areas in different units that allow my students to use different intelligences. This has led to my reputation as the

teacher “who does projects”. I strongly believe there is no panacea for learning. Multiple approaches where students are able to create an artifact that demonstrates their mastery of a subject is far superior to fill-in-the-bubble tests.

This school has a long history of having high academic expectations while addressing social and emotional needs. The challenges are typical of that of an urban, low-income, second-language school. This school has worked successfully with this population for one hundred thirty-eight years and will continue to do so for future generations.

My Rationale

With the bulk of our population coming from homes where English is not spoken, mathematics is often the subject where the students achieve academic success. Traditionally at my school, mathematics scores exceed the reading scores on standardized tests. The last several years, in particular, I have had students who were extremely talented in mathematics and I have been able to work on material that has exceeded sixth grade standards and expectations. One methodology that has aided in this success has been Singapore Math Model Drawing. Model Drawing has allowed my students to identify the parts, wholes, knowns, and unknowns and see the bigger picture when solving problems in mathematics class.

I became aware of my current class of students two years ago when I was working with one of the fourth grade teachers. Knowing her students were well below grade level in mathematics, we worked on different approaches to improve achievement. Her frustration at their lack of progress was memorable. Last year, the fifth grade teachers expressed concern over this same group of students’ low achievement in mathematics. When our ISAT scores in mathematics returned over the summer, the depth of how low this group’s achievement in mathematics on the ISAT became critical. *For the first time in my twenty-three year career, my students have higher reading scores than mathematics scores.* Their average is a significant part of why our school is back on Probation with the city. With a class average of only 45% in mathematics, this is the lowest group in the school from third through eighth grades. On the ISAT, no students Exceed Standards, nine out of twenty-one students Meets Standards, nine are Below Standards, two are Academic Warning, and one I do not have scores for because he transferred to our school from another state. As expected, most of my students do not like mathematics. There is a general feeling of defeat in the room when it is time for that subject.

Algebra has taken on a more important role in the sixth grade curriculum through the Common Core. Number fluency in particular has been a handicap across the classroom. With my students struggling with the basic skills, how can there be success in a component of mathematics that relies on students understanding abstract ideas and how they relate to one

another? A sound foundation laid in sixth grade aids my students into the more abstract mathematics of seventh-grade curriculum.

However, there is a greater urgency than improving mathematics skills, and that is disposition towards mathematics. When it comes to facing challenges, my students have adopted the flight, not the fight, mentality towards mathematics. Repeated failure has impressed upon them the futility of 'getting up off the mat' in mathematics. Mathematics and its role in everyday life has yet to be realized. If this mind-set is not altered, mathematics proficiency will not be achieved. This is less about a grade, a test score, or being prepared for the next school year than it is about valuing a set of skills that is necessary for successful functioning in the world. How does one simultaneously change a disposition and improve ability? Born from these concerns is my Action Research question and sub questions.

Literature Review: What the Experts Say

The trend in mathematics education in the elementary school has changed significantly in the last twenty years. The breadth and depth of what sixth graders are asked to do in mathematics today can be, and is, overwhelming for students who are achieving at grade level proficiency. What about students who struggle with the basics in mathematics class? What tools and strategies are available to them that will provide them entry into higher order mathematics, especially in the domain of algebra?

Algebra: And then Satan said, 'Put the alphabet in math'. (author unknown)

Algebra is often referred to as the gateway to college. More and more high schools are requiring students to pass Algebra I as a graduation requirement (Welder, 2012, p. 255). This emphasis has led to algebra becoming a part of the elementary school curriculum, with many schools offering Algebra I in the eighth grade. Why has this branch of mathematics taken on a more significant role in education? "Because of advancements in the use of technology and its prevalence today, a greater understanding of the fundamentals of algebra and algebraic reasoning is viewed as necessary for all members of society" (National Research Council, 2006, p. 279). Most people would identify algebra as a series of letters and numbers that they either 'got' or didn't 'get'. The struggle to understand the notation of algebra often interferes with the deeper understanding of what is actually occurring with those letters and numbers. However, it is the notation that allows for the reasoning about how operations work. Russell, Schifter, and Bastable argue that "this reasoning – about how numbers can be put together and taken apart under different operations –not the notation is the central work of elementary students in algebra" (2006, p. 3). It is the reasoning through use of notation that should be the central focus of early algebra learners.

“Learning barriers for algebra may result from misunderstandings in students’ existing knowledge,” (Welder, 2012, p. 256). Something as standard as the meaning of the equal sign is regularly misinterpreted by students to mean “the answer comes next” as opposed to representing a relationship between the two quantities on either side of the equal sign. Adding ‘x’ and ‘y’ to either side of the equal sign compounds this even further (Cavanagh, 2009, p. 22). What tools can be identified and utilized in the elementary school classroom to help students overcome the barriers to learning algebra and achieving success in this branch of mathematics?

The Words: Singapore Math Model Drawing

“Man cannot even think without mental images.” – Aristotle

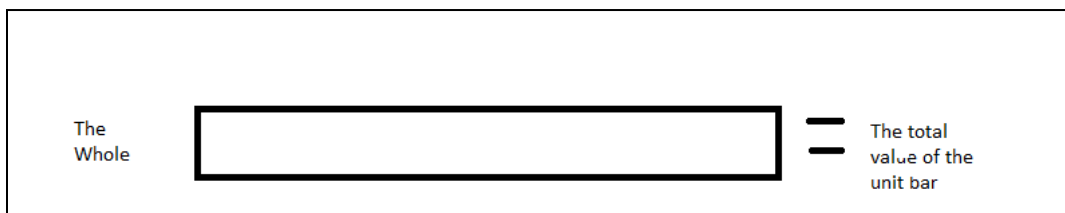
“If I can’t picture it, I can’t understand it.” – Albert Einstein

In 1983, the Singapore Ministry of Education introduced model drawing into its primary mathematics curriculum with the belief that

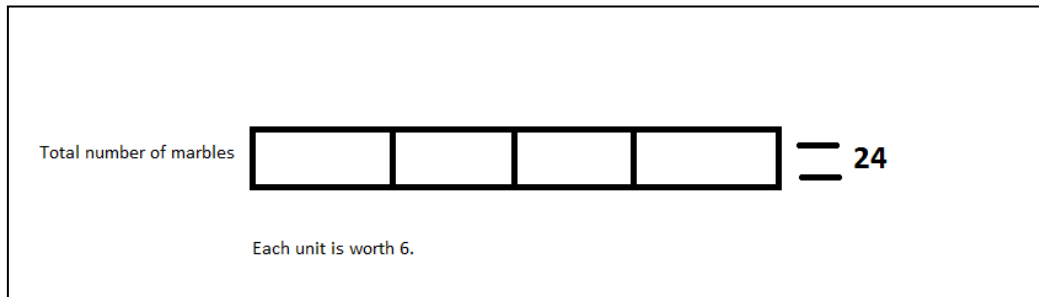
“...if children were provided with the means to visualize a word problem – be it a simple arithmetic word problem or an algebraic word problem – the structure underlying the problem would be made overt. Once children understood the structure of the problem, they were more likely to be able to solve it”, (Ng and Lee 2009, p. 284).

Similar to graphic organizers in the writing process, model-drawing helps students organize the information in a problem in a way that allows them to see the mathematical relationships. The National Research Council’s Adding It Up (2001) refers to this as strategic competence, which is the “ability to formulate mathematical problems, represent them, and solve them” (pg. 124).

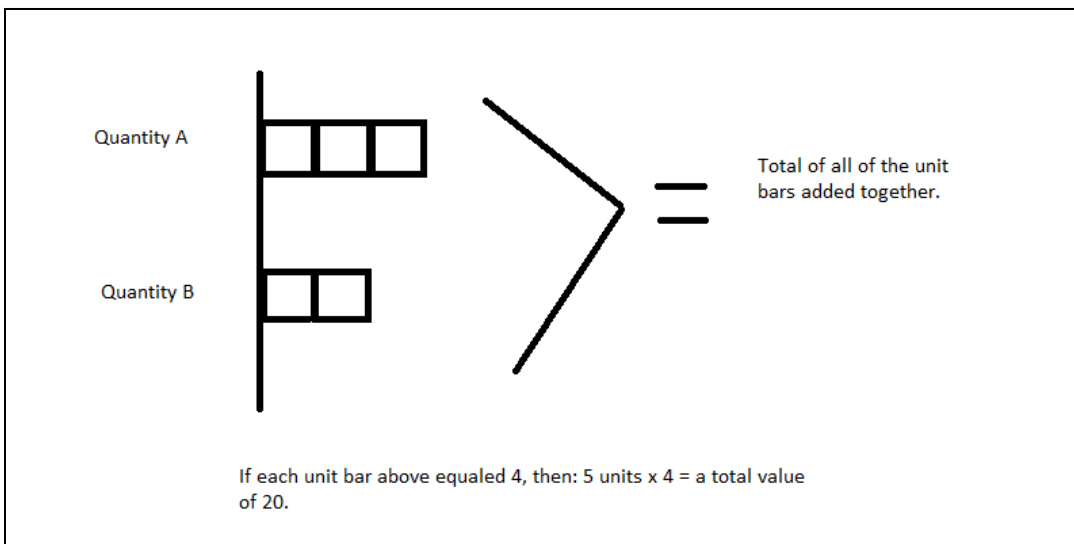
Singapore Math, also known as Bar Modeling or Strip Diagrams, helps students visualize parts of a problem. Each problem begins with a basic visual known as the unit bar. The unit bar is one long unit that is divided according to the actions in the problem. This often falls under the category of Part-to-Whole.



The unit bar represents the total value, or whole, in the problem. The unit bar is then 'cut' to reflect the relationships in the word problem.



Unit bars may also be a series of small bars aligned with one another to demonstrate comparative relationships. These smaller units represent multiple sets of the same quantity. This structure is known as Part-to-Part.

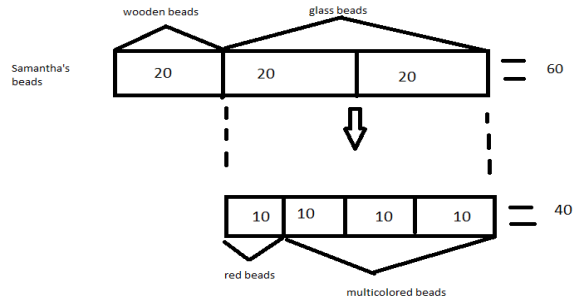


When solving a problem, students are asked to understand the relationship between what is known with the unknown and the actions necessary to bridge the two. For elementary students, the types of problems and their relationships or structures fall mainly into two categories: a) Part-to-Whole, or b) comparison (Part-to-Part). (Ng and Lee, 2009, p. 285-288)

a) Part-to-Whole example

Samantha had 60 beads. Of those beads, $\frac{1}{3}$ were wooden beads and the rest were glass. Of the glass beads, $\frac{1}{4}$ were red, and the rest were multicolor. How many were multicolor? How many were red?

Word Problems for Model Drawing Practice, Level 5, Kuhns (2009)

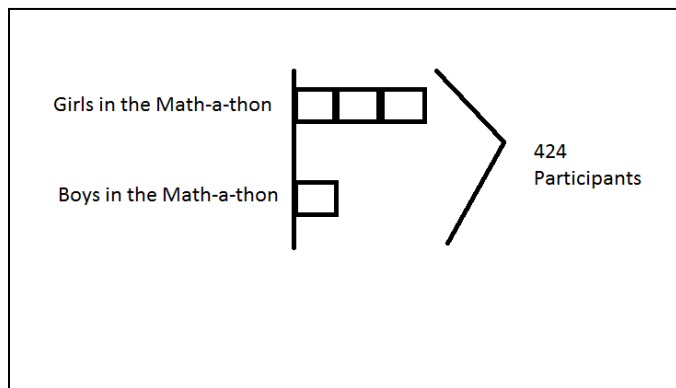


Once students have identified the correct type of problem and drawn a sketch of the model, they then have to label each part of the model. Computation is the last stage of model-drawing.

b) Part-to-Part example

There were 424 participants in the math-a-thon. If there were 3 times as many girls as boys, how many girls participated?

Word Problems for Model Drawing Practice, Level 6, Stipek, (2009)



Each group begins with one bar of equal size. Four units equal 424 participants. In other words four times some unknown (x) is equal to 424 (or $4x=424$). Each unit is equal to 106. The girls have three units, therefore, 3×106 is 318, or 318 girls participated in the Math-a-thon. There are three boxes for the girls because there are three times as many girls as boys in the problem, a very difficult concept for students. This is a basic multiplication problem, yet it highlights a profound misunderstanding students have about the concepts of multiplication.

When instruction has been in the more traditional format, rarely are students asked to identify the structure of the problem (part-to-whole or part-to-part). When students fail to identify which relationship is being addressed, there is tremendous room for error. "The Singapore model method places an understanding on the actions and the relationships in problems," (England, 2010 p. 158).

Students often get lost in the words and, therefore, take whatever numbers are given in the problem and perform the computation that they think is best, usually addition. Being able to identify the structure of the problem, model-draw the structure, label the parts and the whole, and identify the unknown(s) will not only aid in finding the solution, but also provide the scaffolding necessary to build an explanation. “It (model-drawing) communicates graphically and instantly the information that the learner already knows, and it shows the student how to use that information to solve the problem” (Hoven and Garelick, 2007, p. 28). The model also aids students in evaluating their answers to see if they make sense.

The Numbers: Hands-On Equations (The Kits)

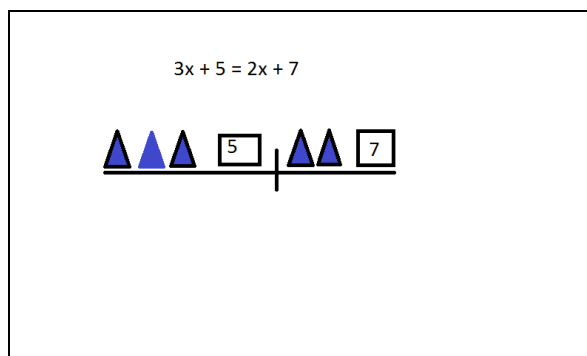
There is a level of procedural fluency that is necessary when solving problems, especially in the domain of algebra. A solid foundation in number facts (addition, subtraction, multiplication, division) is as essential in mathematics as the alphabet and sight words are in literacy. However, the current trend in mathematics instruction does not stress the importance of this fluency. By the time students reach the middle-school grades, their struggles in mathematics increase as their deficiency in number fluency interferes with grade-level proficiency. The use of manipulative materials in the primary grades is a familiar part of instruction. In the middle grades it is regulated to graphing calculators and geo-boards. The importance of using manipulative materials in the middle school should not be discounted.

“Manipulative materials help students make sense of abstract ideas, provide students ways to test and verify ideas, are useful tools for solving problems, and make mathematics learning more engaging and interesting by lifting mathematics off textbook and workbook pages” (Burns, 2007).

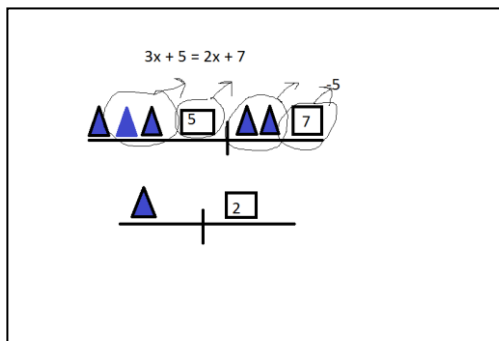
What could be more abstract to a sixth-grader than algebra?

Hands-On Equations, a program created by Dr. Henry Borenson (www.borenson.com), uses manipulative materials to “build links between the object, the symbol, and the mathematical idea they represent” (The National Research Council, 2001, p. 354). Pieces called pawns represent the variables and number cubes represent the constants.

An example: $3x + 5 = 2x + 7$



The initial set-up of the problem has three pawns and a 5 cube on one side of the balance and two pawns and a seven cube on the other side of the balance. By removing the same from each side, students are able to find the value for “x” in a few short steps.



Common Core

In the Common Core Standards for Mathematics, the fourth Standard for Mathematical Practice (Model with Mathematics) states,

“They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.” (Common Core, 2011)

Model-drawing and Hands-On Equations provide a platform for this standard to be utilized in the daily instruction of mathematics students.

The fifth Standard for Mathematical Practice is “Use Appropriate Tools Strategically”, which asserts that a variety of tools, including paper and pencil and concrete models, be enlisted to solve problems.

“Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations” (Common Core, 2011).

Through the use of Singapore Math and Hands-On Equations, students are able to connect abstract ideas to pictorial representations to concrete materials in a way that generates greater success and deeper understanding.

Laying a Foundation

Every morning I have my students begin with a problem of the day. They have a special notebook just for these problems and instruction in this tool is begun on day one. I start off with problems that are one to two grade levels below because I want them to learn how to use the strategy. As proficiency is achieved, the level of the problems is increased. Singapore Math

is also woven into my daily mathematics instruction. The motto of my math class is “When I doubt, draw it out”.

Early in September this class struggled with fourth and fifth grade level problems. By October, they were beginning to master how to set up models that reflected the context of the problems. It was here that their true deficiency became apparent – computation. They would set up beautiful models, complicated models, models that demonstrated understanding of the relationships in the problems, and then add or subtract or multiply or divide incorrectly. In other words, they could do the ‘hard’ part and then get the wrong answer because they never mastered basic computational fluency.

We were heading into algebra and I was seeing my work cut out for me.

Question:

What happens when I implement Singapore Math Model Drawing and Hands-On Equations into algebra instruction with my sixth-grade class of low-ability mathematics students?

Sub-Questions:

- What happens to their attitudes towards mathematics?
- Will it increase their basic understanding of algebra?
- Will it lead to greater success in general mathematics?
- Will participation in math class increase?
- Will it increase facility with math terminology?
- Will students develop an increased ability to self-correct?

Tools for Data Collection:

- 1) At the beginning of the Algebra Unit, my students took a pretest that I designed (Appendix A). They were instructed to do their best, and that I understood that there may be problems on this test that they could not solve. If they really did not have any ideas about how to approach a particular problem, it was okay to leave the problem blank or write “I don’t know” in the space provided. This test will be administered again at the end of the unit.
- 2) I collected NWEA scores in mathematics from fifth grade. I collected NWEA scores from the spring of 2013. I will also collect end of the year scores in May.
- 3) I administered a survey to the students near the end of the unit to evaluate how their dispositions towards mathematics has changed (if at all), during the Algebra Unit (Appendix B).

- 4) I have kept a weekly/every few days journal where I would summarize how the class and particular students are doing in mathematics. I also kept notes of observations made about particular students and any special comments made during mathematics class. I have also kept track of how often particular students are actively participating in class and the nature of that participation.
- 5) I interviewed specific students about their approaches to mathematics and their dispositions towards themselves as mathematicians.
- 6) I have collected student work, math journals, quizzes, and test results in order to follow how my students are progressing throughout the unit.

The Archetypes of Achievement

While working with this particular group of students, I found certain archetypes emerging. They are:

- *The Bookworm* – the kid who loves to read and is always trying to sneak in a paragraph, page, or chapter when they should be working on math instead;
- *The Dory* – the student who ‘just keeps swimming’, even if he/she is not achieving at a high level of success. Don’t worry if you are going the wrong direction, the Earth is round and you will get there eventually; not to be confused with...
- *The Tortoise* – the student who is slowly but surely winning the race. Success has not been by leaps and bounds, but by steady incremental growth;
- *The Ninja* – the student who surprises you with skills you never saw coming;
- *The Calculator* – the student who actually can do calculations accurately, yet struggles with word problems and application;
- *The Conundrum* – the student who can structure the problem, yet still get the wrong answer because of computation;
- *The Scholar* – the student who always has a high level of academic achievement because they work harder than you ever thought humanly possible. (Note: these students tend to be either the youngest or oldest in a family where I have taught all of the siblings. I am sure there is a study on this in a journal somewhere.)

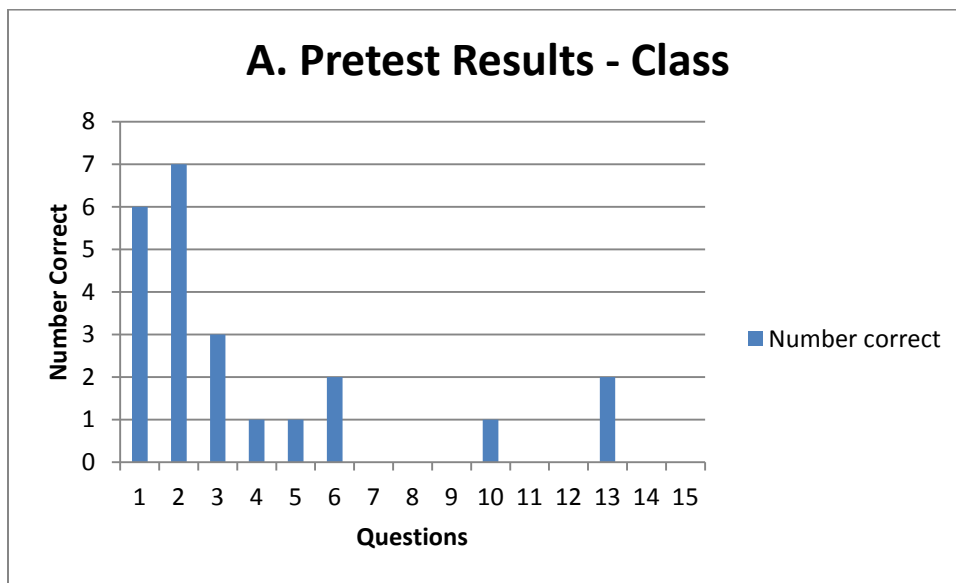
As I review and interpret the data found in this research, I will refer to these archetypes. Each of my sixth grade students emulated one of the archetypes listed above. From time to time, they would veer into other archetypes.

The Pretest

The skills being evaluated on the pretest included: A) identifying variables and constants in a problem; B) comparative reasoning; C) parenthetical expressions; D) distributive property; E)

consecutive numbers; and F) fractional understanding. When I administered the pretest in November, my students had already had exposure to Model-Drawing. The technique had been used in my mathematics instruction in previous lessons. I scheduled 45 minutes for the pretest and let the students know that if they ran out of time, they could finish later, that they would not be penalized for trying. The class was finished in roughly 25 minutes. Only two students worked past the 45 minute mark. Of the fifteen questions on the pretest, nine students did not complete any of them correctly. Four students were able to answer one question correctly, five students were able to answer two questions correctly, and three students were able to answer three questions correctly.

Graph A indicates the number of times each question was answered correctly on the Pretest.



Questions one and two had the highest number correct. Questions seven, eight, nine, eleven, twelve, fourteen, and fifteen were not answered correctly by any students in my class. The pretest was scaled with the easiest questions first and the hardest questions last.

What struck me the most about the pretest results were the number of times “I don’t know” or “IDK” or just plain blanks were on the test. In fact, I saw this 182 times when grading the pretest! Twenty-one Pretests each with fifteen questions results in me evaluating 315 responses. This means that 58% of the questions were not even attempted.

Name	# of times IDK on Pretest	Name	# of times IDK on Pretest
Tiana	12	Keyla	9
Nancy	9	Lupe	2
Yeira	11	Diana	2

Gisselle	11	Paulo	7
Janet	6	Stephanie	7
Jocelyn	12	Angelina	8
Ilenia	7	Mehki	7
Mateo	11	Daveon	10
Ezequiel	11	Emily	6
Vanesa	13	Isaiah	9
Jalen	12	IDK = I don't know	

The Curse of Computation

Computational fluency has been the burden of this particular group of students. Having talked with their previous teachers, this has been a problem for years. No matter how many opportunities this class has had to master number facts, they just won't stick. Is it wrong to call the class Teflon? My room was turning into a class of *Conundrums*! They could often do the more difficult part – set up the problem with the model, organizing all of the pertinent information. They would 'drop the ball' and not get the correct answer because of computational errors.

Examination of the pretest told me that computational fluency was a major factor influencing the problems that were attempted, but not answered correctly. Too often the students demonstrated correct strategies only to get the wrong answer because they didn't add, subtract, multiply, or divide correctly. This supported what I had been seeing in my mathematics class since the beginning of the school year. I had been working on number fluency while teaching a sixth grade curriculum since September.

What was an inconvenience was now becoming catastrophic! How can students be expected to grapple with abstract algebraic ideas when multiplication of two simple digits was proving to be their undoing in a word problem? I had to shift focus on my algebra unit. I had planned on my Action Research focus to be on model-drawing alone. The evidence was telling me that model-drawing alone was not going to be enough to help my students not only improve their algebraic understanding, but also have a positive disposition towards the topic.

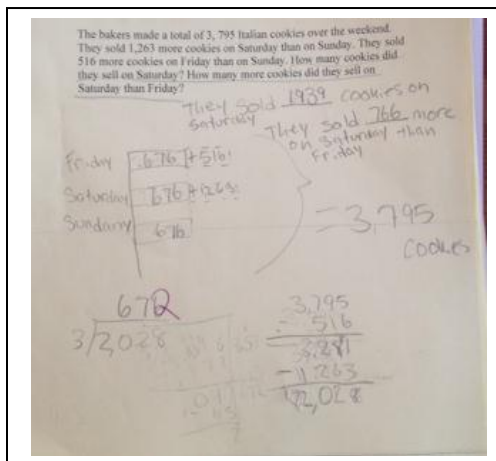
How to get my kids to understand that computational fluency matters? Not an easy task. I know that their previous fifth grade teachers worked like crazy to get the students prepared for sixth grade math. Yet the wall of resistance had already been built. It was time for me to get the hammer and chisel and take that wall down, or at least lower it so they could see above their own limitations and find the value in numbers.

I cannot repeat some of the things said to individuals and the class as they were not always the kindest words. But they were the truth and meant to help them see that what they were doing was not working. Their attitude towards computation was not working. Their defiance towards number fluency was not working. At one point I compared their resistance to tying their shoe laces together and then running a race thinking they were going to win! I took their calculators away. This was the equivalent to taking away an oxygen tank from a scuba diver. They whined. They pouted. They were frustrated. They were mad at me. They said it was not fair. It was taking too long to solve the problem because they couldn't use their calculators. They would be faster, they said, if I let them use their calculators. (Note: I believe calculators to be a powerful tool to aid student understanding. A tool, not a crutch!) Despite their tactics, I continued to stress the value of number fluency. How it can make math actually easier if you have your number facts down cold. I used math games and number races to make it fact fluency fun. I employed manipulatives to make connections to fluency more meaningful. I made time for this because I did not want them to be handicapped by this inability to use number facts.

Hands-On Equations was going to have to play a much bigger role in this Action Research if I was going to see meaningful student growth. In most math classes, algebra is taught as a series of rules and procedures to be memorized. This particular group of students struggle with memorizing basic multiplication facts. The traditional method was not going to work.

Journal 10/11

After weeks of part-to-part I gave my first assessment. EPIC FAIL!!! As usual, many set-up the problems correctly. COMPUTATION IS KILLING THEM!



Looking back at my journal (see above), I had to spend a significant amount of time on Lessons one and two of the Hands-On Equations program. Typically, these lessons are each done on one day. I had to create additional practice sheets for students. In fact, it took six class periods before Lessons one and two were mastered by the class. The Cookie Problem (shown above) is an example of computation getting in the way of accuracy. The student had a perfect model set-up. She used the correct structure, part-to-part. Everything was labeled and the relationships discussed in the problem are accurately reflected. She also understood that in

order to solve the problem, she would have to subtract two constants from the total before she could divide by three to find the value of 'x'. In the far right corner of the paper, you can see she did not subtract correctly. She wrote $3,795 - 516 = 3, 281$ instead of $3, 279$. In other words, she forgot to borrow! This started a chain of inaccuracies that led to an incorrect answer. She knew how to set-up the problem and the steps needed to solve the problem. The Curse of Computation strikes again!

When conferencing with students, I discovered many students struggled with notation. Some misunderstandings included:

- $3x + 4$ to mean 3 x's and 4 x's
- Confusing variables and constants
- $2x$ to mean a 2 and an x

Lessons 1 and 2 Sample Problems	
$X=5$	$2X=8$
$2X+2=8+X$	$4X=3X+5$

And as always, computation was the devil of them. They could often get to where they had variables on one side of the balance and constants on the other. But if there was more than one x, they would often get the wrong result. As I wrote in my journal:

Journal 11/8

I have never had a class struggle with Lesson 2 before. Had to make additional practice sheets. As of last night's homework, 10 out of 21 students cannot complete these problems accurately. I have taken to pairing students together, those who aced the homework with those who struggle. The 'tutors' have to give quizzes after working with their partners on correcting the homework. This is taking far longer that it should. But moving forward would not make it better. If this foundation is not laid now, success will be impossible. I am just so frustrated at this pace! I want to get to the fun stuff. I want to start connecting these ideas to word problems and model-drawing. Guess those will have to go on the back-burner for now. ☹

Something magical happened by the third lesson. Something clicked. As I read my journal notes, students were suddenly getting it. Not only were they getting it, they LIKED the math we were doing in class. Jocelyn (*The Bookworm*), who has no problem letting me know how much she loathes math, stopped me in my tracks when she said "I never thought I would say this, but math is actually fun now!" After I picked myself up off the floor, I asked her to elaborate. Unfortunately, the surly sixth grade girl returned (head down, should shrug, "I don't know" muttered behind a wall of hair, a "please go away, you are making me stand out"

attitude). Yet there were other subtle differences in her attitude towards math. She would ask to pass out the materials at the beginning of class. She would raise her hand to answer questions. She would get to work immediately on the problems when I passed them out. She wanted me to check her work immediately after she finished. She would ask if she could help others in the class because she was finished. All of this from a girl who had to practically have the latest YAL book ripped from her hands because she was trying to read it during math class. She later confessed to tutoring a friend in algebra. I admit it, I had a good time teasing her and have dubbed her my math minion.

11/15
“I never would have thought I would say this, but math is actually fun now!” – Jocelyn (*The Bookworm*)

Many of my students started to ‘wake up’ while using the Hands-On Equations kits. Level I was completed with great results. Eleven out of twenty-one students had a 90% or above, six out of twenty-one had an 80% or above, one student had a 70% average and three students had averages in the sixties. Of my lowest scores, one is being evaluated for special education and two are classic *Dories*.

Tiana, aka *The Dory*, has consistently struggled with math her entire academic career. Her NWEA scores in math in January was a single digit. Yet, I have found her to have some of the greatest growth in the classroom, although her quiz and test scores do not always reflect this improvement. Despite this struggle, she always tries. She has never missed a homework assignment and raises her hand and asks for help. It has been her disposition towards the subject that has been most noticeable. When I asked her if she thought she was better in math, she answered with an emphatic YES. She said,

“Last year was a struggle, it was not my favorite subject. But it is NOW (her emphasis)...now I get it this year because the strategies (Model Drawing) make it easier when I get confused. She related that she “actually like the kits...I understand what the kits mean...they helped with the algebra problems, which are kinda confusing without the kits.”

When working through a lesson, she once described herself as being “math-crazy”. It was time to put the kits away, and she did not want to stop working on the problems. Imagine, a kid who hates math not wanting to stop working on math problems. I believe that if she continues to employ manipulatives and organizers in math class, Tiana will one day find herself achieving great success.

As their computational skills improved with the kits, I continued to weave model-drawing into the curriculum. By the beginning of January, I was seeing some real improvement in their word problem solving skills. They were getting better not only at taking the problems apart and chunking the ideas, but their computational fluency was increasing as well. We were up to an additional ten word problems a week, in addition to the morning math problems, that could be solved using model drawing, the kits, or both.

When the students work on the morning math problem, they are not allowed to check with me if their answer is correct. Instead, they have to find classmates who are also finished and compare answers. If they have different answers, they have to use their work to find the solution. If they have the same answer, they have to find other classmates to compare their work. I only comment on the end, when everyone has had a chance to work with at least one other person. These solution searches can get quite lively. Students will defend their answers passionately and need serious evidence to persuade them if they are wrong. Stephanie, aka *The Ninja*, was turning into a force to be reckoned with in mathematics class. What felt like overnight, but really was the result of months of work, she suddenly was dominating morning math word problems and math instruction. She was usually one of the first correctly finished in the class. When I asked her if she like how morning math was organized, how you have to defend you answer, she responded, "...is very helpful. They (classmates) have their own point of view and you have your own point of view...and then you try to discuss why you have different answers...it is very helpful." This quiet-spoken girl became a fierce warrior when it came to defending her answers. In math class, she went from never raising her hand to having her hand up every time a question was asked. As a fifth-grader, Stephanie had been scored as being on Academic Warning on ISAT. Her January NWEA placed her at grade level. I was excited to see what she would do on the NWEA in the spring.

Slow and steady wins the race. Aesop said so. Many of my students fall into the archetype of The Tortoise. Keyla and Yeira are true *Tortoises*. Both students worked diligently every day on the morning math problems. Both students actively participate in mathematics class. They are active listeners and note takers and homework completers. They are slow to raise their hands to give answers, but are always prepared to answer when called upon. They, too, have struggled with mathematics their academic careers and had poor opinions about math. They do their work because they are good students, not because they find it meaningful or satisfying. The consistency of the strategies has proven to be beneficial. Working on model drawing daily has made the strategy are part of their math 'tool kit'. Keyla related that with model drawing, "I have connected with it a lot. I think because of the fact that now that I have been doing it for a year I now know the strategy more. It kinda helps...but, because I know it now, now when there is a problem, I know when to use it." She said the kits really helped and that she was glad that I bought them. Yeira has a similar arc to her story. She said, "I think I changed a lot (in math this

year) because last year I didn't really like math that much because I felt like I wasn't really good at it. But now I learned more I feel like I got better." She especially liked using the kits because "the answers made more sense". These girls exemplify are a large portion of my class – students with good work habits who needed solid strategies to help guide their problem solving process.

Not all of my students struggled with computation. A few came with the foundational skills necessary algebraic success. Lupe, aka *The Scholar*, is an A student. The youngest of three girls (I taught the sisters), she has always been considered the smartest in the class, according to her classmates. Yet, she is one of the most humble students I have ever taught, humility being a family trait that her sisters also possessed. She and her sisters are first generation American, and the dream of college is already being realized by her older siblings. Lupe's achievement is a result of her extreme work ethic. She struggles with language acquisition from time to time, and figurative language, idioms especially, are often an impediment to deeper understanding. She knows this is her Achilles' heel and works hard daily to overcome any challenges she meets. It is easy to overlook a student like Lupe when doing Action Research. Success is just expected and would only be an issue if she didn't achieve it. So I was surprised during student interviews when she told me how much model drawing and the kits helped. She said last year was harder, especially in fractions. She just didn't get them, but that with model drawing, they now make sense. She also loves morning math when they get to arguing about answers, even when she is wrong! "Having argued for the wrong answer helps me learn even more", Lupe related during interviews. Like I said, she is a special kid.

Daveon, aka *The Calculator*, is another student who came to sixth grade with computational skills. Where Deveon struggles is in reading comprehension. For two years he has been a part of RTI for reading fluency (phonics) and comprehension. If standardized math tests were strictly computation, Deveon would be in the Exceeds category. The language of mathematics has been his handicap. He feels that model drawing "makes the problems easier". Given Daveon's struggles with reading comprehension, I was surprised by his reflection about model drawing. He said, "Model drawing in the morning is kinda fun. It is reading and math together. Model drawing helps with reading and math. The reading is increased because of the morning math. It helps me be a better reader cuz I have to understand the problem." I find it interesting that he feels that math is helping with his reading. As his reading teacher, I can attest to an increase in his reading achievement, especially in comprehension. Further study would be needed to make a direct connection to improved reading comprehension and the model drawing strategy. But HE feels he is a better reader because of model drawing.

Hands-On Equations Test Results

The unit on *Hands-On Equations* has three levels of difficulty. At the end of each level, I administered a teacher-made test to help me evaluate where my students were in the program. Teacher-made quizzes were given throughout each level allowing me to take the temperature of the room.

The results are as follows:

Percentages	Level I	Level II	Level III
100%-90%	11 out of 21	10 out of 21	8 out of 21
89%-80%	6 out of 21	5 out of 21	9 out of 21
79%-70%	1 out of 21	1 out of 21	1 out of 21
69%-60%	3 out of 21	4 out of 21	1 out of 21
50% - 0%	0 out of 21	1 out of 21	2 out of 21

Level I included variables and constants on both sides of the equations. Level II introduced negative variables. Level III has both positive and negative variables and positive and negative constants. The level of computational fluency needed increases with the addition of negative integers. My students did quite well when it came to using the manipulatives in the kits. However, negative integers were quite a challenge, even for *The Scholars*. A great deal of time was spent on manipulating negative integers and still more time will be spent in the future on this skill. Eighteen out of twenty-one students had a 70% or greater average on the Level I test. The three with the lowest averages includes two *Dories* and one student who I am trying to have evaluated for Special Education. Sixteen out of twenty-one students had a 70% or greater average on the Level II test. Although the number of students increased who were below 70%, given the complexity of the mathematics involved, I am relieved the number was not greater. As for the one student who failed the Level II test, he was truant most of the months of December and January. Eighteen out of twenty-one students scored 70% or above on the Level III test. As this Level contains the most advanced computation, I was quite excited by the results. My lowest scorers are either in Special Education or need Special Education services. I was especially pleased with how my *Dories* did on this test. One had a 95% and one had an 80%. This is not to say that their struggles are over. I feel this proves that with the right strategies and time given to master material, success can be achieved by almost all.

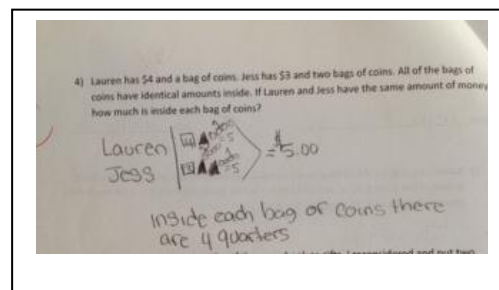
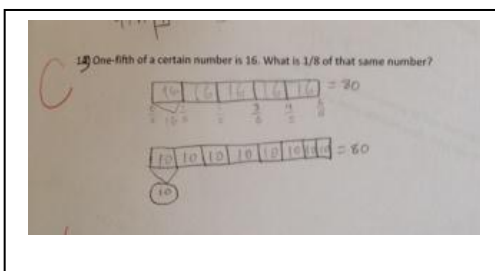
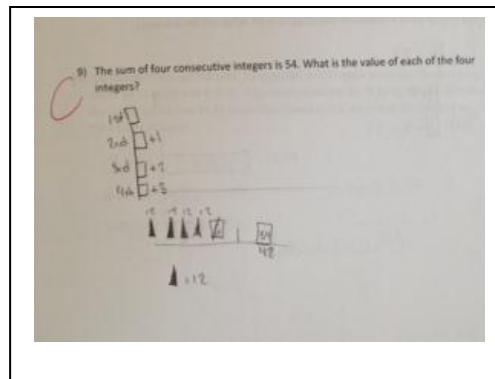
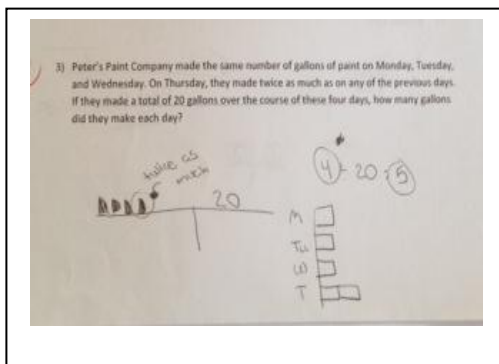
The Post-Test

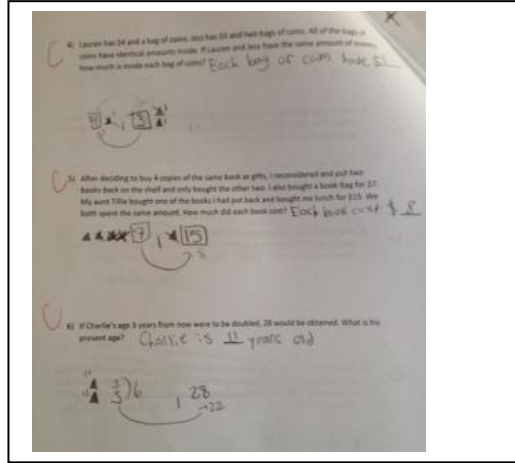
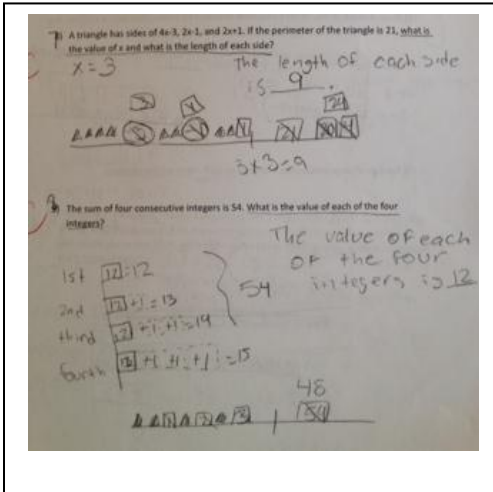
We had finished all three levels of the Hands-On Equations program. We had been solving algebraic word problems for months. It was time to administer the PostTest.

The posttest results were extraordinary, not only in improvement in mathematics skills, but also in effort. As indicated earlier, most of the students did not attempt even half of the problems on the pretest. The table below demonstrates the significant change in student effort.

Name	# of times IDK on Pretest	# of times IDK on Posttest	Name	# of times IDK on Pretest	# of times IDK on Posttest
Tiana	12	0	Keyla	9	0
Nancy	9	0	Lupe	2	0
Yeira	11	0	Diana	2	0
Gisselle	11	0	Paulo	7	0
Janet	6	0	Stephanie	7	0
Jocelyn	12	0	Angelina	8	0
Ilenia	7	1	Mehki	7	2
Mateo	11	0	Daveon	10	0
Ezequiel	11	0	Emily	6	0
Vanesa	13	0	Isaiah	9	0
Jalen	12	2	IDK = I don't know		

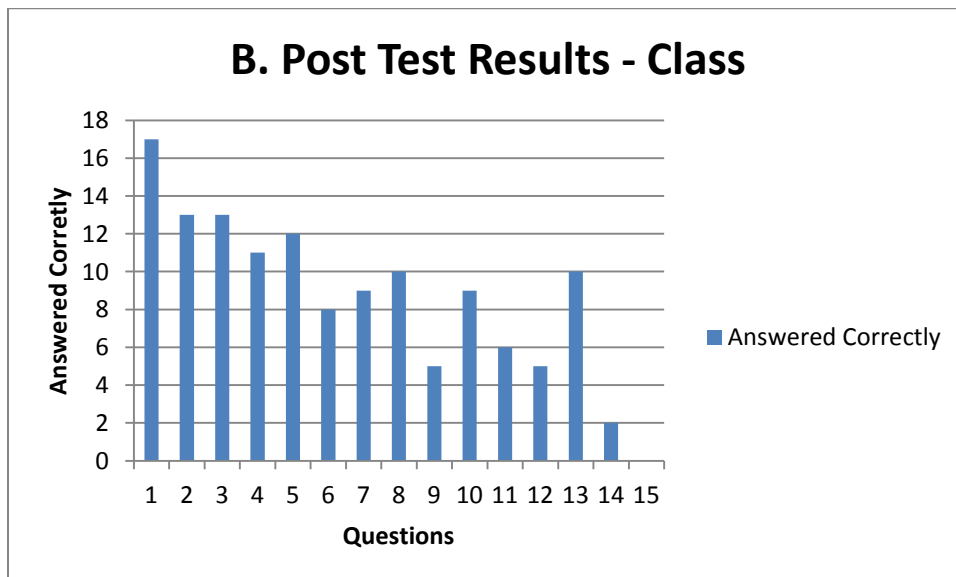
Most of the students attempted to answer all of the problems on the posttest. A variety of strategies were utilized and, in some cases, multiple strategies were employed to answer a particular question. My students used model-drawing, Hands-On Equations, proportional reasoning, and pictures to help them make sense of the problem. Below are student samples of work from the Post Test.



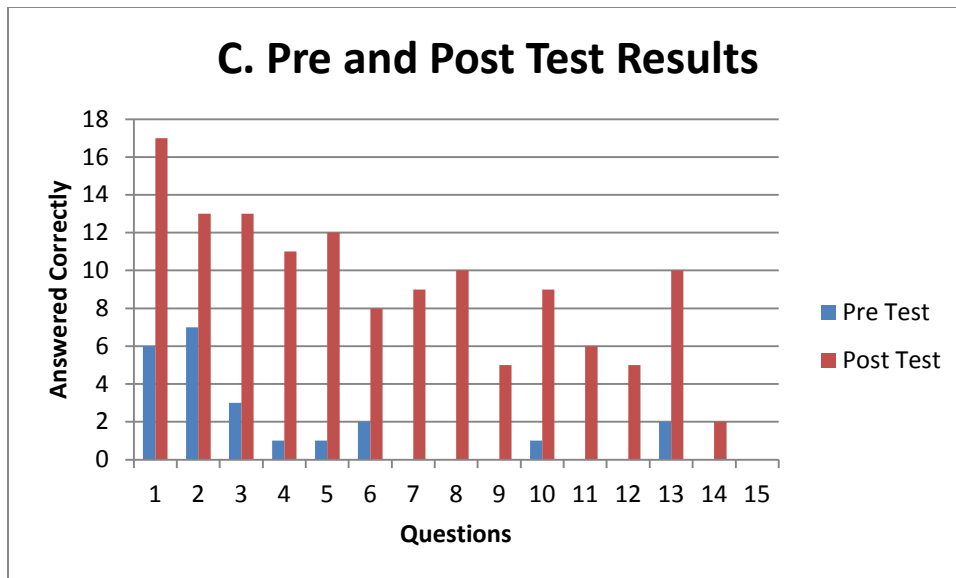


As I look at this work, I can follow my students' thinking through the strategies they employed. Often, students would employ more than one strategy to solve a problem (above left, second problem). Results like this are a dream come true for me. Understanding which strategies to use and when to use them are the height of mathematical reasoning as well as a representation of meta-cognition at its finest.

The Graph B indicates the number of times each question on the Posttest was answered correctly. Not only did the students attempt to answer more questions, they also answered more of the questions correctly.



A side-by-side comparison (Graph C) demonstrates the dramatic improvement in student effort.

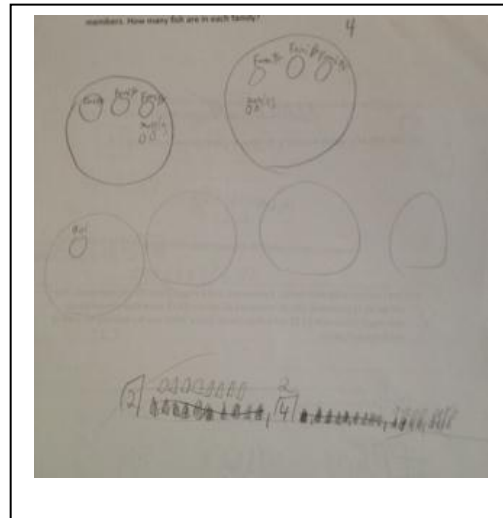
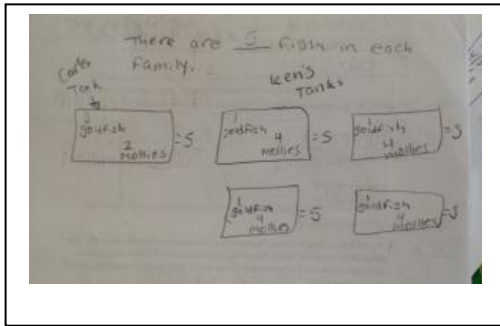
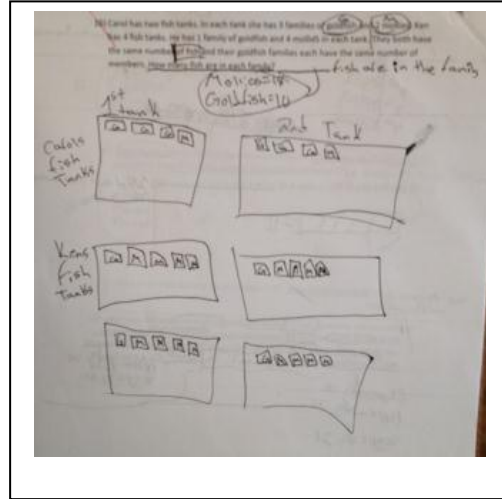
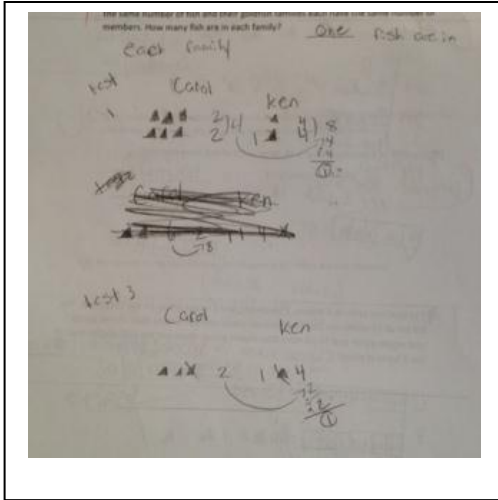


Only the last question, fifteen, was not answered correctly by anyone. Examination of student work indicates that language may have played a part in understanding the problem. Many students attempted to solve this problem with pictures. In most cases, the pictures were accurate in reflecting the main ideas of the problem. As with all the problems, the students employed at least two strategies that were practiced daily, model-drawing and the use of the *Hands-On Equations* kits. Having these strategies embedded in their daily instruction gave them a level of confidence to at least attempt to solve the problems. Often, their errors were computation driven (their Achilles' heel). However, models, pictures and diagrams utilized on the post-test shows me they are on the right path to understanding. Proficiency is just around the corner.

Problem #15 was more challenging because of the word 'family'. As the samples below demonstrate, the students used a variety of strategies that were taught throughout the program. Their work suggests that they have the same value for mollies and families, failing to see that the mollies were a set quantity and it was the number in each family that was the variable. Despite the fact that no one answered this question correctly, I am thrilled with the results. Each student attempted to answer this question utilizing the tools given to them. What more could I ask for (except, of course, for the correct answer 😊).

Problem #15: *Carol has two fish tanks. In each tank she has 3 families of goldfish and 2 mollies. Ken has 4 fish tanks. He has 1 family of goldfish and 4 mollies in each tank. They both have the same number of fish and their goldfish families each have the same number of members. How many fish are in each family?*

Student samples:



What stuck out the most during the posttest was how serious the students were when they took it. It took several class periods for them to finish the test. I swear I could hear them **thinking** as they took the test. The scratch of pencils and the scrape of erasures were the only sounds in the room. There was neither fidgeting nor shuffling of feet. The intensity of their attention to the task was palpable. They knew they knew more than when they started and they wanted to show it, not just to me, but to themselves.

All but two student demonstrated improvement on the posttest. Of those two students, one has an IEP and the other is in RTI.

Name	# correct on Pretest	# correct on Posttest	Name	# correct on Pretest	# correct on Posttest
Tiana	0	2	Keyla	0	6
Nancy	2	7	Lupe	2	13

Yeira	0	7	Diana	1	5
Gisselle	0	5	Paulo	3	6
Janet	3	11	Stephanie	3	8
Jocelyn	1	11	Angelina	2	11
Ilenia	0	7	Mehki	0	0
Mateo	0	3	Daveon	1	6
Ezequiel	0	0	Emily	0	4
Vanesa	1	11	Isaiah	2	4
Jalen	2	4			

Ezequiel has an IEP that includes consultation minutes for mathematics. He is also the classic *Bookworm*. I have had to wrestle more books out of lap during mathematics class than I care to count (no pun intended). I am currently building a file on Mehki to receive Special Education Services. It is unfortunate that he made it to sixth grade without having raised the alarm bells. I believe the CPS policy of only retaining students in third, sixth, and eighth grade may have played a role in this situation. He passed third grade with the lowest score possible on ISAT and the gap has been widening ever since. It is extremely difficult to get older students into Special Education. He will be attending Summer School, at the end of which CPS will determine if he is to go on to seventh grade, which I am strongly against. Also, our Network has placed a HEAVY emphasis on reading with almost no interest in mathematics. We do not have a formal RTI program in place for mathematics because our funding has been focused on reading and language arts. This lack of interest on the Network's part in mathematics has left kids like Mehki with no place to go for support.

The successes on this Post-Test were, in some cases, quite dramatic. Many students more than doubled the number of correctly answered problems. Some vanquished a dragon! Students like Jocelyn, Vanesa, Lupe, Keyla, Angelina, Daveon, Yeira, Stephanie, Ilenia, Gisselle, and Janet came to show algebra who was boss. They had strategies and confidence on how to use those strategies. They wrote, scratched out, erased, and started over again. They tried one strategy and then another when the first did not work. They took their time. They paid closer attention to computation. They permitted themselves to be active, thinking problem-solvers. They were on offense instead of defense. They were brave.

NWEA

There is no escaping standardized testing. As the results from the ISAT, which has traditionally been the measuring stick for CPS, are not available for this year I will be examining my students NWEA or MAPS test results. A RIT score indicates that the student was answering questions

correctly 50% of the time at that level of difficulty. The students' expected growth for this school year was six points, with 225 being the magic RIT for sixth grade. As you can see in the chart below, most of my students exceeded the expected six point gain. Only one student went down on his RIT score. He finished the test in about twenty minutes, whereas the rest of the class took over an hour to complete the exam. His scratch paper was blank. When I asked him what happened, he said he thought he knew it. Pride cometh before the fall.

Name	Spring 2013 NWEA	NWEA 2014 Goal	Spring 2014 NWEA	+ or -
Tiana	194	200	211	+ 17
Nancy	215	221	233	+18
Yeira	213	219	226	+13
Gisselle	207	213	226	+19
Janet	225	231	229	+4
Jocelyn	206	212	225	+20
Ilenia	206	212	220	+14
Mateo	230	236	223	-7
Ezequiel	201	207	217	+16
Vanesa	225	231	244	+19
Jalen	205	211	212	+7

Name	Spring 2013 NWEA	NWEA 2014 Goal	Spring 2014 NWEA	+ or -
Keyla	203	209	215	+12
Lupe	232	238	240	+8
Diana	208	214	223	+15
Paulo	229	235	238	+9
Stephanie	205	211	227	+22
Angelina	207	213	235	+28
Mehki	194	200	209	+15
Daveon	221	227	235	+14
Emily	201	207	207	+6
Isaiah	NA	NA	244	Unavailable

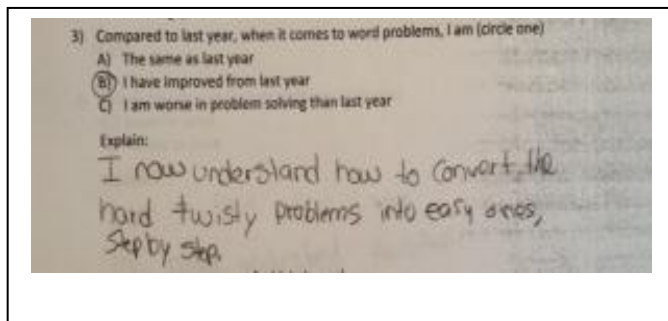
I do not believe one test on one day is a true measure of my students' abilities. I do not feel this reflects their growth in their mathematical understanding nor does it reflect their changing dispositions towards mathematics. What this does show me is that I am on the right track regarding the use of consistent strategies being utilized in the mathematics program. The kind of growth demonstrated above is quite staggering. I knew my students grew, but to see

these kinds of numbers made my heart skip a beat. I admit that when Mateo finished first, my stomach dropped with his score. It would be almost another forty minutes before the rest of the students' scores started coming in. For forty minutes I sat wondering if I had done more harm than good by focusing on strategies. I was vindicated (or rescued, depending on how you look at it) by the rest of the class! Their scores were available immediate after they finished the test. There was a great deal of pride being felt by my sixth graders. Each student (except Isaiah, he transferred from another state) was able to compare last year's score to this year's score. They knew they not only met, but exceeded, the goal set by NWEA.

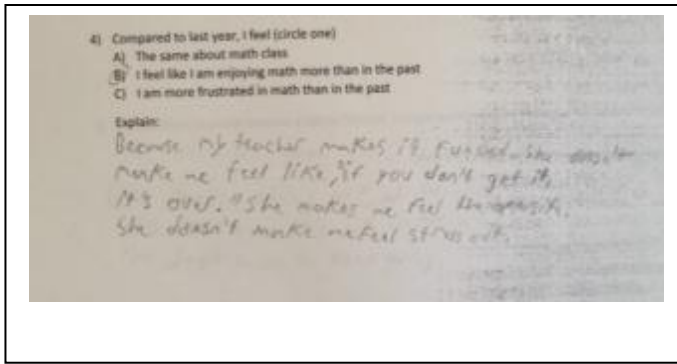
A note about the goals that NWEA sets: Each student is given a specific goal to be met on the Spring Test, which is based on the score from the previous Spring Test. Meeting the goal does not indicate grade level ability. In other words, they can meet their goal and still be several grade levels either above, or more often, below grade level. As their score determines promotion in CPS at sixth grade, it is confusing for students to see that they met their goal but still have to attend Summer School.

About those Dispositions Towards Mathematics...

Towards the end of the unit on algebra, I gave my students a survey about how they felt about themselves as math learners and problem solvers (Appendix B). Overall most students felt they were stronger in math. They also felt more empowered when it came to solving algebra problems because they knew how to use the kits and Model-Drawing. All of the students indicated that they felt there were better at problem solving this year than last year. They all indicated that model-drawing and the kits made problem-solving easier. All but three feel they are enjoying math more than in the past. The three who did not circle that option circled that they felt the same about math class this year as last year. All students selected that their computation skills have either improved or stayed the same. No students felt that their computation skills have decreased this year. All but three students agreed with the statement "that you can learn to be more successful in math". The three who circled "you are either good in math or bad in math" wrote: "I think I am good in math." "I am improving." "I think I am doing better in math because I have gotten grades than last year." These explanations lead me to believe that they didn't quite understand the question, that, in fact, they are improving.



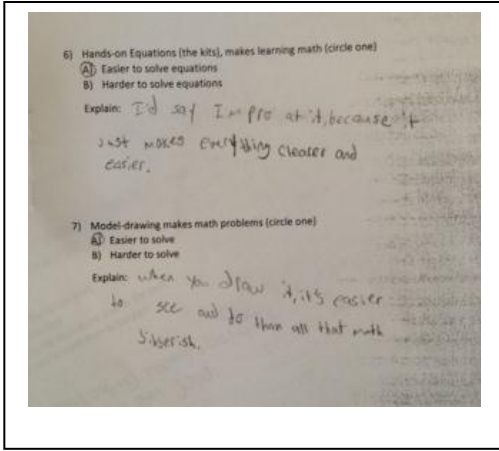
"I now understand how to convert the hard twisty problems into easy ones, step by step."



"Because my teacher makes it fun and she doesn't make me feel like, "if you don't get it, it's over." She makes me feel like the opposite, she doesn't make me feel stress out"

"I'd say I'm a pro at it, because it just makes everything clearer and easier."

"When you draw it, it's easier to see and do than all that math gibberish."



One insight the survey provided for me was how important my own disposition towards mathematics and problem solving was in their success. Time and again they mentioned how my belief that math was important and valuable made them feel that it was worth learning. Food for thought.

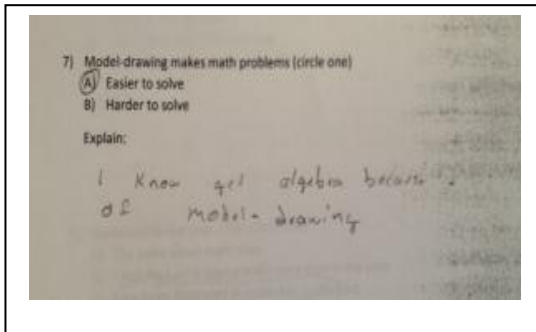
Conclusion:

I currently have a *Bookworm* and a *Scholar* going head-to-head in morning math. On some mornings I swear I hear the sound of a boxing bell ring when they are trying to work through the problem and get to the correct answer first. They are quite serious in their efforts to take each other down and win the title of Supreme Math Being. These are two girls do not run in the same circle. Yet this friendly, if not fierce, competition is the result of a confidence found when mastery of a strategy meets computational fluency. It is a wonderful way for me to start my day.

No one strategy works for every student. I still have students who cannot subtract two digit numbers involving borrowing and renaming, especially the renaming. I still have sixth graders who struggle with dividing 45 by 3. Equations with fractions make the wheels come off the cart for almost all of my sixth graders. We walk through them together, compare them to similar problems with whole numbers, but still, a code red is sounded. Subtracting negative integers is still a challenge. Most of my students can get the first part of a problem organized

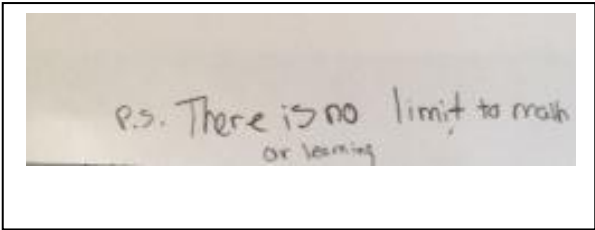
through Model-Drawing. Many can finish the model and find the answer, although I still have students failing on the computational side of the problem.

However, a consistent use of strategies can aid students towards mastery. I feel this action research has further confirmed for me the power of Model-Drawing and *Hands-On Equations*. As strategies go, they have aided my students to better ‘see’ what the problem is asking of them. For some of my students, there is still a long road ahead to mastery. Other students are well on the way to an Algebra I course. It is nice to know I have provided them with some tools to help them on their journey.



“I know [now] get algebra because of model-drawing.”

“P.S. There is no limit to math or learning.”



The above statements confirm for me that I am on the right path as well.

Policy Recommendations

Network Level

- The Network must continue to provide mathematics professional development across the grade levels.

School Level

- As a staff, we will develop a set of mathematics problem-solving strategies, including manipulatives, to be threaded throughout the grade levels.
- The school must dedicate RTI minutes to students struggling in mathematics classes.
- The school must set aside professional development specifically for mathematics instruction.

Classroom Level

- The teacher will consistently use a set of problem-solving strategies on a daily basis.
- The teacher will model enthusiasm for mathematical practices.
- The teacher will reinstate manipulatives for struggling students.

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- 4) Lauren has \$4 and a bag of coins. Jess has \$3 and two bags of coins. All of the bags of coins have identical amounts inside. If Lauren and Jess have the same amount of money, how much is inside each bag of coins?
- 5) After deciding to buy 4 copies of the same book as gifts, I reconsidered and put two books back on the shelf and only bought the other two. I also bought a book-bag for \$7. My aunt Tillie bought one of the books I had put back and bought me lunch for \$15. We both spent the same amount. How much did each book cost?
- 6) If Charlie's age 3 years from now were to be doubled, 28 would be obtained. What is his present age?

7) A triangle has sides of $4x-3$, $2x-1$, and $2x+1$. If the perimeter of the triangle is 21, what is the value of x and what is the length of each side?

8) The sum of four consecutive integers is 54. What is the value of each of the four integers?

9) The greatest of three consecutive integers is 56. What is the sum of the three integers?

10) Three students in Ms. Smith's class have birthdays on consecutive days in October. The sum of the dates is 51. What are the dates of their birthdays?

11) The sum of three consecutive odd integers is 87. What are the integers?

12) The sum of four consecutive even integers is two more than five times the first integer. What is the value of each integer?

13) One-fifth of a certain number is 16. What is $\frac{1}{8}$ of that same number?

14) The teachers ordered 6 cheese, 3 pepperoni, and 2 veggie pizzas for the class party. The bill for all 11 pizzas was \$59.75. Pepperoni pizzas cost \$0.75 more than cheese pizzas and veggie pizzas cost \$1.25 more than cheese pizzas. What was the subtotal for each of the 3 types of pizza?

15) Carol has two fish tanks. In each tank she has 3 families of goldfish and 2 mollies. Ken has 4 fish tanks. He has 1 family of goldfish and 4 mollies in each tank. They both have the same number of fish and their goldfish families each have the same number of members. How many fish are in each family?

Appendix B

Survey

1) Compared to last year, my math grades have (circle one)

- A) Stayed the same
- B) Have improved
- C) Are worse

Explain:

2) Compared to last year, when it comes to computation (+,-,x,/), I am (circle one)

- A) I am the same as last year
- B) I have improved my skills
- C) I am worse in computation

Explain:

3) Compared to last year, when it comes to word problems, I am (circle one)

- A) The same as last year
- B) I have improved from last year
- C) I am worse in problem solving than last year

Explain:

4) Compared to last year, I feel (circle one)

- A) The same about math class
- B) I feel like I am enjoying math more than in the past
- C) I am more frustrated in math than in the past

Explain:

- 5) I believe (circle one)
- A) You are either good in math or bad in math
 - B) That you can learn to be more successful in math

Explain:

- 6) Hands-on Equations (the kits), makes learning math (circle one)
- A) Easier to solve equations
 - B) Harder to solve equations

Explain:

- 7) Model-drawing makes math problems (circle one)
- A) Easier to solve
 - B) Harder to solve

Explain:

- 8) Do you think you have become a better 'thinker' in math word problems? Explain.