Number Talks Reasoning and Creativity in Middle School Math Classrooms

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QUESTION

What happens to my students' number sense when I implement regular number talks?

SUBQUESTIONS

- What happens to my students' ability to solve a problem in multiple ways?
- What happens to my students' conceptual understanding of arithmetic and their ability to explain their reasoning?

I. CONTEXT

Goudy Elementary is a public school located in Chicago's Uptown neighborhood. It is an open-enrollment school and serves students from pre-kindergarten through eighth grade.

Goudy's student population reflects the diversity of Uptown, a home to many recent immigrants. Among the students, 29 different languages are spoken, and 40% of the approximately 750 students speak English as a second language. Most Goudy students also fall below the poverty line, with 93% qualifying for free or reduced lunch.

Goudy staff and students take pride in the progress our school has made over the years. In 1988, then Secretary of Education William Bennett declared Chicago Public Schools the worst in the nation, singling out Goudy as the worst school in America. The negative publicity spurred the Illinois General Assembly and Goudy into action. Because of strong leadership and localized control, Goudy made an incredible turnaround, and, by 2002, became a success story for neighborhood schools. The high standards set by former principal Patrick Durkin during the turnaround persist today. Now, under the leadership of Principal Pamela Brandt, Goudy is a level 1 school in good standing, known for its diversity and its integration of technology.

In my nine years as a Goudy teacher, the integration of technology has been a priority. Through grant-writing and fundraising, each classroom is equipped with an interactive whiteboard, each student in grades five through eight has access to an iPad during the school day, and lower grades have laptop carts. Students receive education in coding and media arts. This year a group of fifth through eighth grade students are participating in a maker's lab, in which they create and 3D print models of their work.

There is a strong sense of community and professionalism among Goudy teachers. They regularly attend professional development together on their own time and form professional learning communities, including a teacher book club and peer observation group.

The Goudy staff stays connected to the Uptown neighborhood through community events and partnerships with local organizations. Our third grade students take a weekly field trip to read with the residents of a nearby senior citizen home. Members from the community regularly volunteer in our school garden or visit the school to read on our monthly "Reader's Friday" event. This fall, teachers and students participated in the Edgewater 5K run to help raise money for our school.

Our principal also works with nearby schools. This past summer, after severe budget cuts were announced, teachers at Goudy and McCutcheon Elementaries cleaned and painted each other's buildings. Mrs. Brandt also initiated an intraschool teacher kickball tournament to build community among the neighborhood schools.

II. RATIONALE

"Number sense —"a person's fluidity and flexibility with numbers" (Gersten and Chard, 1999)—is critical to higher-level mathematics. Without it, algebra, geometry, and calculus are beyond a student's grasp. But teachers themselves often struggle with number sense and how to teach it. Many resort to timed math drills to build competence and fluency. I have tried several ways of including math drills into my lessons. Rather than building understanding and fluency, this practice seems to reinforce the idea that people are inherently good or bad at math. Students who know their math facts speed through the exercises. Others get frustrated as more students finish ahead of them. The fast-finishers rely on memorized procedures, and the struggling students or slow problem solvers are left feeling like failures. All of the students are aware of who does and does not know their facts. No one learns anything.

Math instruction that values being quick and right over reasoning, flexibility, and creativity results in students who have a shallow, procedure-based understanding of mathematics. Students finish their work and, happy to be done, never question whether their answers make sense. As elementary- and middle-school teachers, we aim to develop number sense in our students, as it is essential for success in mathematics.

Poor number sense takes many forms in my middle school classroom. Confident students, who quickly answer a computation question, become quiet when asked to explain their thinking. Some offer long, rambling explanations of a procedure they learned years ago, but cannot explain if or why it makes sense. Students freeze at the sight of a word problem and wait silently, hoping someone will tell them what to do. Students who rely mainly on formulas and tricks reach for calculators when asked to estimate. These students do not understand the relationships between numbers and lack flexibility in solving problems. I was curious if number talks would help my students be more creative and thoughtful in their problem solving.

III. LITERATURE REVIEW

Here, I briefly review the literature regarding number sense, number talks, and a teacher's role in number talks. This broad overview traces my own exposure to this are of pedagogy.

A. Number Sense

While often confused with memorization of math facts, number sense is the ability to think and reason about quantities, numbers, and formal symbols, such as written numbers and operation signs. According to Burns (2007), “Students with a strong number sense can reason flexibly with numbers, use numbers to solve problems, spot unreasonable answers, understand how numbers can be taken apart and put together in different ways, see connections among the operations, figure mentally, and make reasonable estimates.” (p. 24). Because number sense is the foundation for all higher-level mathematics, misconceptions stemming from a lack of number sense become more pronounced as students progress through elementary and middle school.

When we equate number sense with memorization of facts, we value only the procedural aspect of mathematics and we deny students the chance to apply logic and reasoning skills in ways that make sense. To illustrate the difference between procedural and conceptual understanding, consider my knowledge of cars. I have a procedural knowledge of my car. I can start it, put it in gear, drive it, and park it. If something goes wrong with my car, I cannot diagnose or fix the problem. I have to take the car to the experts: my mechanics, Gary and George, who have a procedural and conceptual knowledge of cars. Many students have a procedural understanding of numbers, but lack conceptual knowledge. They can compute quickly and accurately, but have trouble determining a reasonable solution or diagnosing when a procedure does not work.

Students who demonstrate flexibility with numbers are more successful in mathematics than those who have only memorized rules and procedures (Gray and Tall, 1991). Numerical fluency goes beyond fast recall of math facts and algorithms. Students develop number sense by working with numbers in different ways and communicating their reasoning with others (Boaler, 2008). For example, students who rely on memorization and procedure will immediately use the standard “borrowing” method to solve the subtraction problem *23 - 19*. While these students may do the algorithm correctly and get a correct answer, they also demonstrate that they are no longer thinking about the actual numbers (Humphreys and Parker, 2015).

Mathematics is procedural, but it is also visual, creative, and open to different methods. Students who prioritize memorization of facts and procedures become less likely to think about numbers or develop number sense (Boaler, 2009). Number talks engage students in reasoning about mathematics and build number sense by allowing them to think more deeply about numbers and patterns.

B. Number Talks

Number talks are short, purposeful discussions designed to transform students’ mathematical reasoning (Humphreys and Parker, 2015). They aim to engage students in problem solving and reasoning as students find and discuss different ways numbers can be calculated (Boaler 2008). During a number talk, students mentally solve carefully selected computation problems, then talk about their strategies. The teacher serves as recorder and facilitator. The key components of number talks include a collaborative classroom community, classroom discussions, the teacher's role as a facilitator, the role of mental math, and purposeful computation problems (Parrish 2011).

In a typical 5-to-15-minute number talk, students prepare by putting away paper and pencils. Students solve the problem mentally, putting up their thumbs when they have reached a solution. When most thumbs are up, the teacher asks for volunteers to share their answers. All answers, correct and incorrect, are written on the board. The conversation continues as students share and discuss different solutions, strategies, and ideas while the teacher listens and asks guiding questions.

A positive classroom community is essential for effective number talks (Parrish, 2011). Students have to feel safe in order to contribute their ideas to the discussion. Teachers should establish an environment in which: (1) Students explore many ways to see and do problems; (2) Students are responsible for communicating clearly; and (3) Students are responsible for trying to understand other people's thinking (Humphreys and Parker, 2015). Teachers and students encourage multiple viewpoints, and mistakes are celebrated as learning experiences. If used consistently, number talks can help create a safe community in the classroom. Because the discussions are student-centered, number talks provide regular opportunities to practice positive group interactions. With time, patience, and consistency, number talks can create a learning environment based on acceptance and common learning goals (Parrish, 2011).

While the structure of number talks will vary, students, not teachers, are the primary participants. They break numbers apart, reconstruct them, and explain their reasoning clearly to others. In the process, students develop a deeper sense of numbers, as well as the ability to think flexibly about problem solving and communicate their reasoning(?). For adolescents, this opportunity to be involved in reasoning gives them an important sense of ownership in their learning (Boaler, 2009). Rather than being told by the teacher what makes sense, they discover how to make sense of mathematics.

C. The Teacher's Role in Number Talks

The goal of number talks is for students to think and reason deeply about mathematics. They need time and practice to develop number sense. To successfully implement number talks, teachers should become comfortable with wait time, ask open-ended questions, encourage students to be clear in their explanations, and be open to multiple strategies (Humphreys and Parker, 2015). Teachers must also commit to regular number talks. To be truly effective, number talks must be a frequent and consistent part of a math class (Humphreys and Parker, 2015). As a formative assessment tool, it helps to listen and be genuinely curious about what students understand. Recording students' methods and helping them rephrase their thinking gives students a chance to participate in their learning.

Quality math instruction develops students' ability to think, reason, and solve problems. (Burns, 2014). While number talks are student-centered, the teacher sets the purpose for the number talk and guides the discussion toward the intended purpose. The teacher's role during number talks is that of facilitator, recorder, questioner, listener, and learner (Parrish, 2011). For teachers who are used to explaining procedures and guiding students to a correct answer, facilitating number talks can be a steep learning curve. With daily practice and reflection, the quality of the number talks will improve over time (Humphreys and Parker, 2015).

IV. NUMBER TALK DEVELOPMENT AND DATA COLLECTION

In this section, I first discuss how I developed number talks in my classroom. I then examine the various ways—audio and photographs, formative assessments, student surveys, and teacher journals—I collected data regarding that development.

A. Developing Number Talks in My Classroom

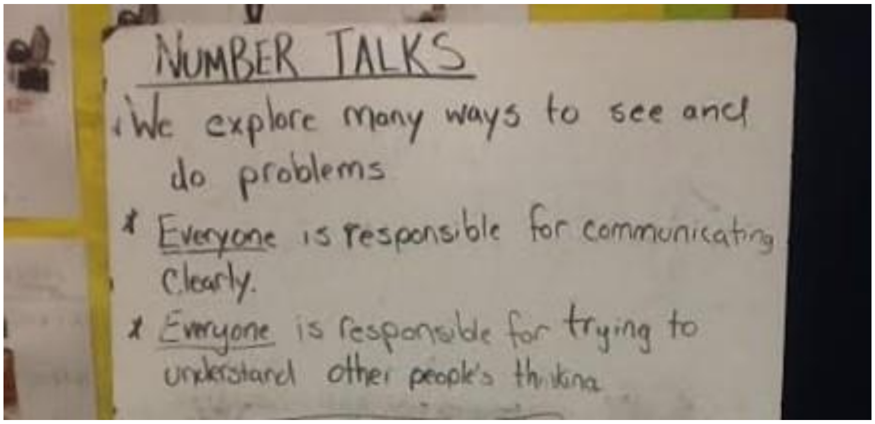
Number talks developed in my classroom over the course of the year. While my research centers on a single sixth-grade class, I used number talks in all four of my sixth- and seventh-grade math classes. I documented my learning process using my teacher journal. Despite needing a small amount of class time, number talks required careful planning of procedures, discussion techniques, problem selection, and student assessment.

The need for a positive environment became evident early on as I wrote in my October 14 journal:

> While most of the conversation went well, there was one problem I didn't anticipate. Jessica loves to participate, but is really sensitive about mistakes. When another student asked her to clarify her explanation, she got defensive and angry. As a class, we discussed how everyone is responsible for communicating clearly and trying to understand each other.

During the first month of school, I built a safe environment for the students to share ideas in number talks. I began with routines and procedures. On September 24, I wrote, "I chose to bring the students to the rug in order to separate the number talk from the rest of the class. I also wanted to make sure the students were visualizing their thought processes without writing anything down."

That week, I introduced a signal to the students that there would be a number talk. If the lamp in the classroom library was on, they knew to begin class on the rug. I posted three expectations: (1) We explore many ways to see and do problems; (2) Everyone is responsible for communicating clearly; and (3) Everyone is responsible for trying to understand other people's thinking (Humphreys and Parker, 2015). I referred to the expectations frequently to guide our conversations and redirect students when necessary.



This focus on signals, routines, and procedures made number talks novel, helped build our classroom community, and gave the students a sense of stability.

At first, I used number talks about twice weekly. One month later, I increased them to four times per week in order to boost participation. After that, I wrote in my journal, "I'm noticing a lot of progress in the quality of the conversations we are having. Students are participating and paying attention. They are asking really great questions and seem personally invested in the conversations." After experimenting more with different durations and frequencies, I held number talks three to four times per week, limiting them to 15 minutes.

Throughout the year, I used my journals to study my questioning techniques, monitor my use of wait time, and reflect on how I could use number talks to help students make sense of numerical relationships. I struggled with motivating all students to participate regularly in our discussions. About 20% of the class would only participate if called on. Time management was also a concern, as some number talks could last up to 30 minutes. I eventually used a timer to end each discussion after 15 minutes.

The number talks were a powerful formative assessment tool in my classroom. I used observations from number talks to be more responsive to students' needs and learning styles. My journals from these conversations revealed information about students' fundamental knowledge of numbers. On October 14, I reflected on a number talk based on the problem 43-9:

> Arianna and about five others used the standard algorithm in their heads. Sharon and several other students counted backwards by ones, some of them using their fingers to count. Saad broke apart the subtrahend, subtracting 3, and then 6. Ixomara split 9 into 3 groups of 3, then subtracted in 3 equal "jumps." Jessica used the standard algorithm, but added up to check her work.

This helped me know which students were less confident with manipulating numbers and which were reliant on procedures. Although I used this strategy as a formative assessment, I never graded my students on their answers or responses in number talks. I felt that assessing my students for a grade would detract from my listening to the content of their conversations.

Number talks were now fully integrated into my daily classroom routine, and most students look forward to them. Students seemed to feel a sense of ownership during the talks;

they have their own spots on the rug, their own pillows, their own strategies, and their own questions. This helped me build community in my classroom and gave me a unique way to know my students as creative problem solvers.

I began using regular number talks with my middle school math students in September 2015. My data collection took place over four cycles, each representing a different mathematical concept addressed in the number talks.

B. Audio Recordings and Photographs

Beginning in December 2015, I made audio recordings of number talks and transcribed the conversations. I also photographed the board I used to record students' answers and strategies. I used the audio recordings and photographs to record data about the quality of conversations and the level of student participation. These two methods also provided insight into student reasoning, vocabulary, flexibility, and confidence.

C. Formative Assessments

I issued a formative assessment at the beginning and end of each data collection cycle. Each assessment asked students to attempt to solve a problem in two different ways, explain their reasoning, and identify their level of confidence in their work. These assessments allowed me to examine students' flexibility in problem solving and reasoning skills, as well as identify patterns of understanding or misconception across the class.

I used formative assessment data to analyze students' conceptual understanding, as well at their ability to explain their reasoning. I looked for ways in which students demonstrated their understanding using academic vocabulary, visual representations, or real-world examples. I used this data to highlight the differences between students conceptually based and procedurally based students(?).

This data also provided a way to analyze students' problem-solving flexibility. I asked students to attempt two different ways to solve a problem similar to the problems we discussed in number talks.

D. Student Surveys

I administered a weekly student survey. In it, I asked students to self-assess their learning and participation in number talks. Students identified how often they participated in number talks and answered questions about the strategies they were learning and using. I used this data to gauge student participation and identify which strategies were most helpful to students.

E. Teacher Journals

I used my teacher journals to reflect on the general progress of the class regarding number talks. Throughout the year, I recorded my observations about the quality of students' questions, explanations, vocabulary usage, and participation. I also used my journal to reflect on the challenges my students and I faced in each data cycle.

V. DATA ANALYSIS

In this section, I test my questions and subsquestions: (1) What happens to my students' ability to solve a problem in multiple ways? ; and (2) What happens to students' conceptual understanding of rational numbers and their ability to explain their reasoning?

A. What happens to my students' ability to solve a problem in multiple ways?

Students demonstrate flexibility in problem solving when they are able to solve a problem in multiple ways, break numbers down, use estimation effectively, and select efficient problem-solving strategies. Developing flexibility with numbers requires creativity and deep thinking about numerical relationships.

I approached number talks both as a mathematical and creative exercise. One of our early number talks was not about numbers, but about art. I asked my students to look at a photograph and tell me what they saw:

Me: *What do you see when you look at that picture of a wave?*

Issac: *That's a wave? I thought it was a hole.*

(Several more students express surprise that this is a picture of a wave)

Me: *That's awesome. Isaac, you thought it was a hole. Saad, you thought it was the Earth.*

Jessica: *I thought it was just water and color*

Kerry: *I thought it was just a bunch of paint.*

Several other students offer ideas

Me: *So we are all looking at the same thing and yet we're all seeing something different. Our number talks will work like that too. We will look at number problems and talk about the different ways to solve them. What's different with numbers is that you've mostly been taught only one way to see and solve problems. Here you can come up with your own ideas. There's a bunch of different ways to show that if you add four and four you get eight. You've learned one way. What we are doing here is finding other ways to see it in* *your own way like you look at that picture. That's why we're doing this.*

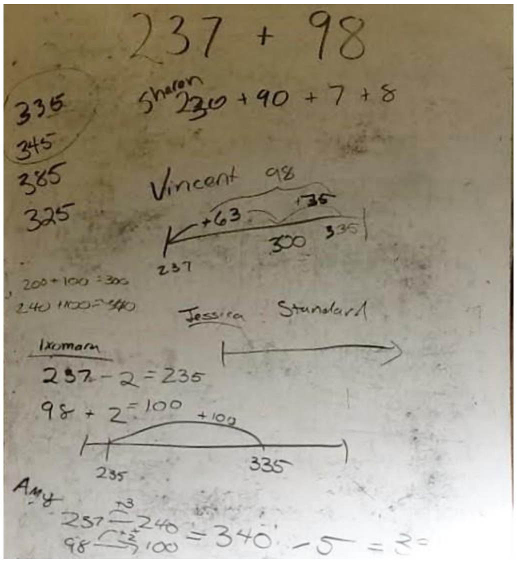
As number talks continued through the year, many students developed their own ways of solving problems and viewing numerical relationships. The table below shows formative assessment data from cycles one through four. In this formative assessment, students were to solve the problem using two different methods, show their work, and explain their reasoning. It represents data from 24 students in one sixth-grade class.

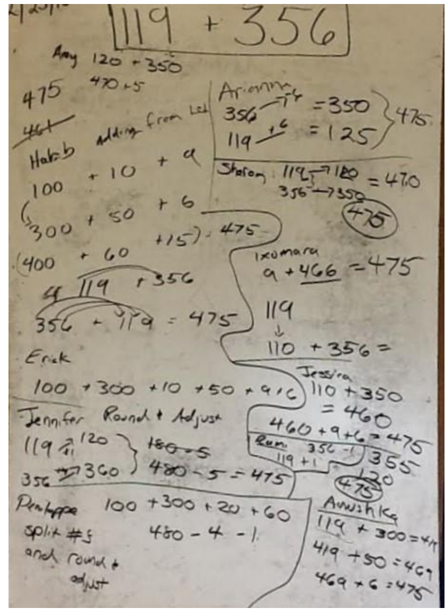
Formative Assessment Data								
Cycle/ Topic	Cycle 1: Fraction Benchmarks (Closer to 0, ½, or 1)		Cycle 2: Comparing Fractions		Cycle 3: Multiplication		Cycle 4: Addition	
	Beginning	Ending	Beginning	Ending	Beginning	Ending	Beginning	Ending
Successfully used more than one method	32%	42%	18%	42%	50%	86%	46%	87%

The data show an increase in some students' ability to solve a problem using multiple strategies. Some students gained more flexibility in their mathematical thinking and problem solving during each data cycle. The most dramatic changes in flexibility occurred in addition and multiplication problems, perhaps because these operations are more accessible to students than problems with fractions. Between the beginning and ending of each cycle there is a gain in the number of students who use multiple strategies. This gain increases with each cycle. For example, in Cycle 1 there was an increase of 10%; by Cycle 4, there was a gain of 41%. This suggests how some students are more confident in trying different strategies.

i   Multiple Strategies for Addition

During the first addition number talk, I observed four students (about 17% of the class) counting on their fingers. I also noticed seven students (29%) drawing in the air with their fingers. This was an indication that they were attempting to solve the problem using standard procedures. The pictures from an early number talk (left) from this cycle compared with a number talk that took place 4 days later (right) show the difference in the number of strategies students tried.





Transcripts from the number talk on the left show that 14 students attempted to solve this problem mentally using the standard algorithm. This picture also shows that we had three incorrect answers. Most of the students who used the standard algorithm made a mistake when trying to carry a number mentally. This picture also shows that students introduced some new methods for addition. Ixomara uses a "take and give" strategy, in which she takes 2 from 237 and gives it to 98 before adding. Her new problem, 235 + 100 is easier to solve. Sharon adds the numbers from left to right, Amy rounds both numbers then adjusts her answer, and Vincent breaks apart the second addend. This excerpt from the transcript shows how I prodded my students to think about which method will work best for them:

Me:	*Look at the strategies on the board. Please choose one that you think would require the fewest steps. Which one is most efficient? Efficient meaning which one can you solve without doing a whole lot of work.*
Kerry:	*I did standard, but I think Ixomara's method. It looks easier. She just subtracted 2 and added two.*
Ashley:	*I like Ixomara's and Amy's because they made the problem easier to do in your head.*
Christian:	*I like how Sharon broke the number down into smaller numbers.*

The second photograph shows how more students developed their own ways to mentally solve addition problems. Transcripts from this day show that only three people used the standard algorithm. The photograph also shows how students were using strategies from the number talks in their own ways. For example, Habib, Erick, and Anushka all added from left to right, but broke up the numbers differently. Habib and Erick break up both addends and added them by place value (300 + 100; 10 +50; 9 + 5). Anushka broke up only the second addend (119 + 300 + 50 + 6). These students constructed their own ways of thinking about numbers.

ii Multiple Strategies for Multiplication

The multiplication number talks revealed that many students were dependent on standard algorithm to solve a problem, yet had little understanding of multiplicative relationships. As students progress through school, they are taught to mainly rely on the standard algorithm for multiplication as shown below:

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  1
  16
 x12
  32
+160
 192
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Students sometimes (or feel?) powerless to determine if an answer is reasonable. In the beginning of this data cycle, students were rigid in their procedures. In the first number, talk I observed more than half the class drawing in the air with their fingers. While most students used the standard algorithm, I recorded nine incorrect answers on the board. After two number talks on this subject, several students began to develop new methods for solving the problem mentally, but many still felt comfortable with the standard algorithm. One student's frustration is demonstrated in the following conversation:

Jessica: *I don't get how these people get these strategies. They look at it and are like, like all these math people are like, "I'm gonna do that, I'm gonna do this, I'm gonna multiply this and divide that and subtract that and here's my answer." I just know it's 9 times 7* is 58.

Me: *How many other people feel like that, like "I don't know where these strategies come from." All I know is the standard way."?*

[Several hands go up]

Me: *How do we get these strategies?*

Ixomara: *You have to look beyond what you see. You see 7 x 9, I see 3x3x7.*

Me: *So it has to do with how you see how the numbers are made up?*

Ixomara: *Basically, you gotta um. You gotta think about it a different way from the way they taught you. They taught you that 'cause that's* *the basics. You gotta eventually try and find out some other ways to solve it.*

This conversation illustrates how dependence on a single strategy can be harmful to a student's mathematical growth. Jessica was agitated and became defensive about her reliance on math facts for fast answers. She was, in fact, so concerned with a fast answer that she did not notice her incorrect answer, 58. In these number talks, we explored new ways to solve multiplication problems, as well as methods to check if solutions made sense.

The first formative assessment (23 x 9) revealed that 50% of students could not solve a multiplication problem without the standard algorithm. Of those that could, three students used the inefficient method of repeated addition, and one student drew a diagram of 23 rows of 9 boxes.

Throughout cycle 3, I chose problems that would be increasingly difficult to solve mentally using the standard algorithm. I wanted to lead the students to discuss when it might be useful to know another strategy. The following conversation about the problem *16 x 12* is typical of our early multiplication number talks:

Me: *I saw a number of us writing with our fingers. Were you using the standard algorithm too?*

Few kids: *Yes*

Me: *What did you guys think about that method for this problem?*

Vincent *It was hard, but then I figured it out and did it another way*

Jessica: *I got it wrong with the standard way because I needed a pencil*

Me: *So this becomes tough to do mentally*

Arianna: *It's hard to keep track of all the numbers doing this mentally*

Over the next several weeks, students discussed new methods to multiply numbers mentally, while I represented their work on the board. Students developed methods using area models, factoring, breaking numbers into addends, using the distributive property, halving and doubling factors, rounding and adjusting, and even combining several strategies. The following excerpt from our number talk using, as well as the photograph, shows how the participating students became more flexible and creative in their problem solving. The students discussed the problem *81 x 25*. Before the discussion of multiplication strategies, they eliminated unreasonable answers. These are shown in the photograph as numbers that are crossed out:

Napat: *I did 81 time 5 and got 405, then I multiplied 405 by 4 because I already used one five.*

Jessica: *Napat said that he already used one five, but…*

Napat: *Oh! I want to change my answer to 2025 because I should multiply 405 by 5*

Sharon: *He broke up the 25 into factors, He used one five to multiply by 81, then used the other five to find the answer. He did 5 times 5 times 81.*

Me: *So it looks like Napat factored a factor. Who did this another way?*

Ixomara: *I was thinking that 25 times 4 is 100, and I used what Saad said about it having to end in a multiple of 25, so I multiplied 25 times 80 and got 2000. Then I multiplied 25 times 1 and got 25. Then I added it together and got 2025. I was confused, but then I used Saad's method.*

Me: *How would you describe Ixomara's method?*

Jennifer: *I have a question. Why did you use the 25 again after you used it when you multiplied by 80?*

Ixomara: *That was the factor I didn't change. I changed the 81 to an 80 so I had to multiply the 25 by the 1.*

Me: *Why does this make sense?*

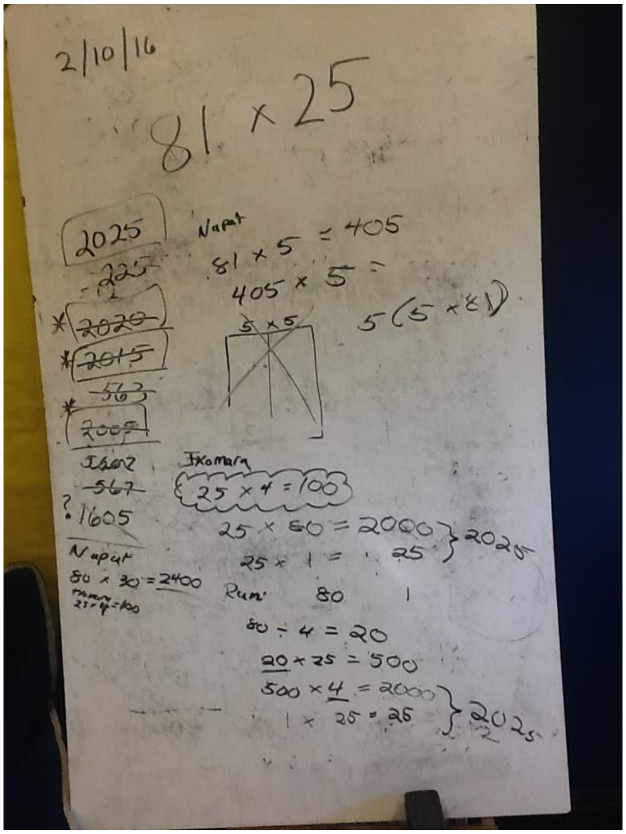
Habib: *It makes it an even number*

Jessica: *It makes it easier to multiply than 81*

Rami: *I got 2025. I made the 81 into 80. Then I divided 80 by 4 and got 20. I multiplied 20 by 25 and got 500. Multiplied 500 by 4 and got 2000. Then I added the other 25 and got 2025.*

Vincent: *Rami split up the 80 into 20 times 4. He multiplied 20 by 25 then by 4. Then he multiplied 25 by 1.*

Amy: *He rounded then broke up the 80 into factors to make it easier to multiply.*



The data collected from the number talks and the final formative assessment in Cycle 3 indicate that many students were developing several methods to solve multiplication problems. The number of students who relied only on the standard algorithm to solve multiplication problems decreased by 29%. This data also shows that some students have learned from each other how to break apart numbers, put them back together, and identify when an answer is reasonable or not.

B. What happens to students' conceptual understanding of rational numbers and their ability to explain their reasoning?

In this subsection, I explain how my research shows changes in students' ability to correctly can solve a problem and explain their reasoning.

i. Reasoning with Fraction Benchmarks

Students with strong number sense have a conceptual understanding of numbers that extends beyond performing procedures. They can solve a problem, determine if their answer makes sense, and explain what they did. By listening to the number talk discussions, I evaluated what each student understood about numerical relationships. The number talks also provided insight into patterns of misconceptions among the whole class. Faulty reasoning and misconceptions were especially noticeable when we studied fractions and decimals. In a journal from November 30, I wrote about my students' struggles with fractions:

> Many students could not either answer or provide a reasonable explanation about whether a fraction was closer to 0, 1/2, or 1. Almost all students were successful at identifying if a fraction is greater or less than 1/2."
>
> In one example, a student was asked to identify if 9/16 was closer to 0, ½, or 1. The student replied, "It is closer to 1 because half of 16 is 8 and 9/16 in more than 8/16." Many students believed that if a fraction was more than ½, it had to be closer to 1.

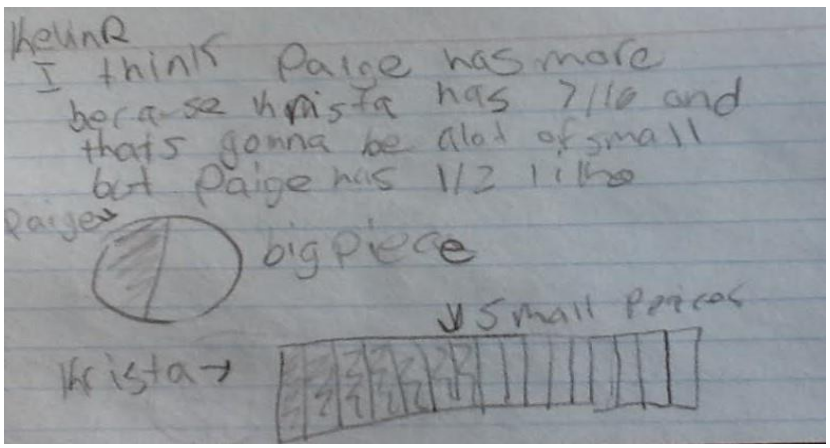
Similarly, on February 29, students solved the problem *5.3 +0.9*. I noted in my journal that students would "go to great lengths to not deal with decimals." A student explained his method, "I took out the decimal, then added 50 plus 9 plus 3. I got 62, then I put the decimal back in for six point two." While this answer is correct, it indicates a fundamental confusion about decimals, which usually leads to students "putting the decimal back" in the wrong place.

Particularly, where fractions and decimals are concerned, a rote procedural approach to teaching mathematics can lead to a superficial understanding. Many of the rules that work for whole numbers do not work for fractions or decimals. My students carried many common misconceptions about fractions and decimals. I used formative assessments to measure conceptual understanding, as well as how clearly a student could explain their reasoning. The table below contains formative assessment data from all four cycles.

Formative Assessment Data (rounded to nearest whole number)								
Cycle/ Topic	Cycle 1: Fraction Benchmarks (Closer to 0, ½, or 1)		Cycle 2: Comparing Fractions		Cycle 3: Multiplication		Cycle 4: Addition	
	Beginning	Ending	Beginning	Ending	Beginning	Ending	Beginnin g	Ending
Correct answer with sound reasoning	33%	67%	25%	54%	50%	63%	50%	80%
Correct answer faulty reasoning	42%	8%	42%	13%	38%	21%	38%	12%
Incorrect answer	25%	25%	33%	33%	12%	14%	12%	8%

The formative assessment data shows that there was an improvement in some students' conceptual understanding and reasoning skills throughout each cycle. In this first data collection cycle, I studied students' ability to use benchmarks to estimate the value of a fraction. Students were asked to determine if a fraction was closer to 0, ½, or 1. A student who can use benchmarks as tools for estimation shows they understand that fractions represent a part to whole relationship.

I observed that students were reliant on drawing circular or rectangular models of fractions to represent their thinking. Often, their drawings were not the same size, nor were they divided into equally sized sections. For example, in the picture below, Kevin compared two fractions, but used one circular model and one rectangular model. He correctly answered that 7/16 is less than ½, but he reasons that it is because 7/16 is "gonna be a lot of small (pieces) which is smaller than one big piece."



As our number talks continued, we explored other ways of representing fractions. In the final formative assessment, more students used number line diagrams, estimation, and the number's distance from the benchmark to explain their thinking. When asked to place the fraction 17/41, for example, a student responded, "It's closer to half because since 20.5 is half of 41, 17/41 is only 3 spaces away. It will take more spaces to get to one than to get to 1/2." In this way, the student demonstrated an understanding of the part-to-whole relationship of the fraction.

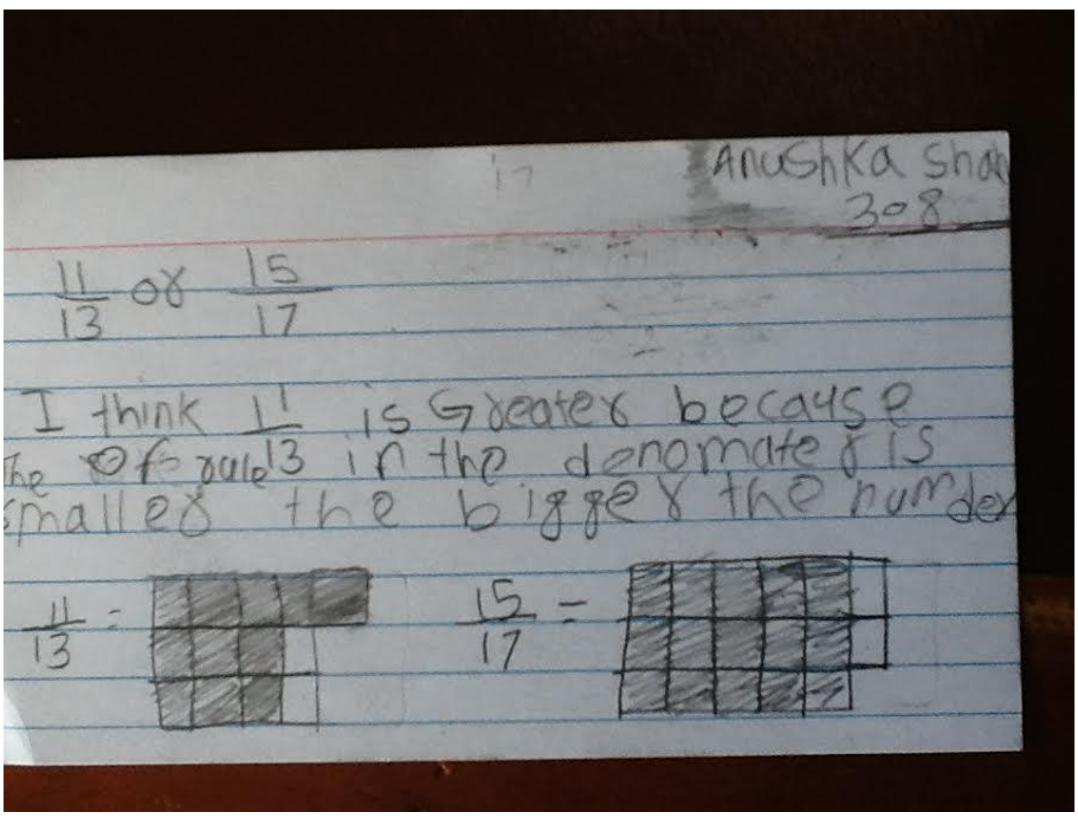
The data shows that, in this cycle, some students made progress in their ability to explain their reasoning, yet the number of students who incorrectly answered the question remained flat at 26%. Of these students, three of them incorrectly thought that the fraction was closer to one because it was more than one half. One of the students attempted to draw a model of the fraction, and one student incorrectly interpreted the problem.

ii. Reasoning with Comparing Fractions

In Cycle 2, students compared the value of two different fractions. Traditionally, students learn to compare fractions by rewriting the fractions with a common denominator, then comparing the numerators. For example, to compare ¼ and ½, one first changes ½ into 2/4. Because 2/4 is greater than ¼, ½ is the greater fraction.

Beyond explaining the steps to a procedure, a student with a conceptual understanding of fractions is able to explain why one fraction is greater than another using the part-to-whole relationship of a fraction. When looking for evidence of conceptual understanding, I looked for students to explain the number of pieces a fraction is divided into, refer to the size of the pieces, or the distance of a fraction from a benchmark on a number line.

While some students in Cycle one students grew in their conceptual understanding of fractions using benchmarks, this did not immediately transfer to comparing the values of two fractions. Although 65% of students were able to correctly use a benchmark to gauge the value of a fraction, only 23% could clearly explain how they compare the values of two fractions. In the first formative assessment, Arianna incorrectly stated that 3/6 and 7/15 are "probably equal to each other because I drew diagrams and they are both equal lengths." Arianna's diagrams were different-sized rectangles divided into unequal pieces. Other students reverted to procedural methods that they could not explain. Jessica multiplied cross-products, but could only explain the procedure, not why it works. This indicated a deeper conceptual misunderstanding of fractions. Anushka's work, below, is representative of the misconceptions many students have about fractions:



Her models do not show a whole divided into parts. Rather, both models show a number of equal-sized parts, determined by the denominator. The shaded pieces represent the amount in the numerator. Like many other students, Anushka sees fractions and whole numbers as unrelated.

Our number talks in this cycle focused on the part-to-whole relationship in fractions. I wrote in my teacher journal that about 75% of the students used the least common denominator method and explained the process. This was within their comfort zone and did not lead to very interesting discussions.

Giving them the fractions 1/8 and 1/7, I asked them to discuss with each other how else they could know that 1/8 is greater. Amy stated, "You can look at the fraction with the smaller number. It will be the larger fraction." The students eventually came up with the idea that, when comparing two fractions that have the same numerator, the fraction with the smaller denominator will be greater. Saad justified this by saying, "the denominator tells you how many pieces the fraction is cut into. If it's cut into 5 pieces, the pieces are bigger then if it's cut into 7 pieces." Two days later, the following conversation and photograph indicated that some students were building their conceptual understanding and ability to explain their thinking:

Ixomara: *I know that both fractions are two spaces away from one and that the smaller denominator means that it's cut into bigger pieces. 9th's are bigger than 13th's so 7/9 is the bigger fraction.*

Amy: *If both are 2 away from zero, the fifteenths are a smaller space away from zero than the ninths, so that means that 13/15 is bigger.*

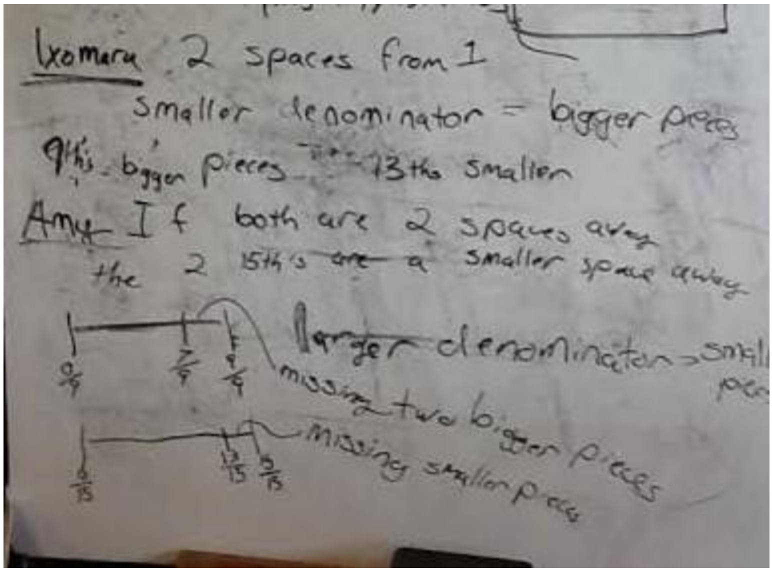
Ixomara: *Wait, can you break that down more for me? I'm trying to understand.*

Amy: *The larger the denominator, the smaller the pieces. Both fractions are missing two pieces, but 13/15 is missing two smaller pieces, than 7/9.*

Vincent: *So if it's on a model, the 13/15 are closer to one because the 2/15's are smaller than the 2/9.*

Ixomara: *Oh!*

Arianna: *I did the same thing that Ixomara did, like saying the ninth pieces are larger than the 15th pieces so 7/9 is bigger. But now I can see, like on a number line, that 13/15 is bigger.*



At this time, other students offered their understanding of the problem. Several students referred back to writing equivalent fractions using 45 as the LCD. Jessica, who was still mostly comfortable with procedures, asked Isaac to help her out with finding the LCD. She was also able to explain using the idea of "smaller pieces" and "closer to one." During this cycle, there was a 29% increase in the number of students who would correctly solve the problem and explain their reasoning. These students showed gains in their conceptual understanding of fractions as parts of a whole number.

C. Limitations

In this section, I explain some of the limitations I encountered with my research including: (1) Students who showed little growth; (2) Participation; and (3) Connecting learning from number talks into the rest of my math class.

i. *The Incorrect Answer Group*

Throughout all four data cycles, the percentage of students who repeatedly gave an incorrect answer remained flat. Although this percentage does not include all of the same students, two students were consistently in this group, showing little or no growth in their ability to correctly solve the given problem. The formative assessments show that these students increased their efforts to solve problems and explain their reasoning.

Ahmed, a recent refugee, had not attended school in five years. He kept to himself and did not speak in class, including number talks. Progress with him took time and individual attention. In the beginning data cycles, Ahmed left his assessment blank, showing no attempt at the problem. His more recent work showed attempts at the problems, although the answers were incorrect.

Erick was new to the school, having immigrated from Mexico in September. He was a motivated and social student, but there was evidence of gaps in his prior knowledge, particularly in math. He could understand spoken English, but preferred to answer in Spanish. In the beginning of the year, Erick remained quiet during the number talks. He attempted formative assessment problems, but solved the problems incorrectly and could not explain his work. Beginning in Cycle 3, Erick participated more in the number talks. He gave his reasoning in Spanish, which was then translated by another student into English. Although he still gave incorrect answers more often than not, Erick was learning how to explain mathematical processes and concepts.

ii. *Participation*

Students who participated the most in number talks will get the most out of them. Yet, I never managed to get all students to participate regularly in number talks. My journal entry from November 24 shows how I tried to engage more students:

> I've started experimenting with different strategies for increasing participation in the number talk. I noticed that students were falling into roles of either participant or observer. Using a written conversation helped more students express their thinking. I also tried having students turn and talk to a partner during the number talks. Finally, I called on new students, but with some forewarning.

Even with these strategies, there were students who participated infrequently or not at all. This was not entirely dependent upon mathematical ability. Data from the student surveys showed that students like Bethani, Jennifer, and Erick shared their ideas regularly, despite frequent incorrect answers and reasoning.

The student surveys indicate that an average of 12% of students never spoke up during number talks in a given week. For some students, this was because of language, for others, shyness. The difficulty of the problem also influenced participation. During the cycles that focused on fractions, the percentage of students who participated infrequently or not at all was between 20-33%. I wrote in my journal about how more difficult problems resulted in less participation while problems that are too easy do not lead to a good discussion. I learned that I must carefully select number talk problems to increase engagement.

iii. *Connecting Number Talks to Math Class*

A problem surfaced during the year that I did not anticipate. My journal entry from January 19 describes students who have not connected their learning in number talks with their learning in math class:

> Although the students are participating and asking great questions during the number talks, this is not always transferring to their work in math class. Our lesson today required the students to solve some double digit multiplication problems. Most students immediately resorted to the standard algorithm. Similarly, they did not return to check if their answer made sense or to prove that it was correct.

I realized that I needed to do more to reinforce ideas from number talks in our daily math lessons, or students would resort to old, familiar methods. Like learning any new skill, this requires frequent practice and encouragement.

VI. Conclusion and Considerations for Teachers

Number talks are generally an effective strategy for encouraging flexibility and reasoning skills in problem solving. They engage many students in conversations about foundational mathematical ideas and allow students daily practice with academic vocabulary. Students who participate develop new strategies for solving problems and gain confidence in their own reasoning. Number talks are easy to incorporate into a daily routine as they require a small amount of time and preparation.

While they need a small amount of planning and class time, number talks require the teacher be reflective and responsive. Throughout the year, I experimented with different questioning techniques, levels of scaffolding, and discussion strategies to encourage my students to participate in and benefit from the number talks. Although my research focused on one class, I used number talks in all four of my sixth- and seventh-grade math classes. I also observed my student teacher as she conducted her own number talks. These experiences helped me develop some ideas and practices for using number talks which may be helpful to other teachers.

A. *This will be different for every class.*

I implemented number talks with four math classes: two sixth grade and two seventh grade. Each class responded differently to the talks, and each required somewhat different strategies for success. The classes with a high level of trust among the students were more confident and more likely to take risks. These students required the least amount of scaffolding and had the richest discussions. For classes in which the students did not get along as well, I applied different strategies to encourage participation and risk-taking. For example, when these students were less willing to share their ideas, I would write out a solution from another class and ask the students to discuss it. I noticed in these classes that the same students were participating. To encourage more students to participate, I would call on random students and ask them to explain each other’s work. In all of my classes, I frequently praised students who shared their mistakes.

B. ***Participation is important.***

Students will not learn from number talks, unless they actively participate in them. Although I was able to get most students to speak up at least twice a week, there were always students who did not volunteer their ideas or questions. These students indicated in the survey that they participate by solving problems in their heads and listening to others explain methods. These students may not have enough practice in explaining their thinking or reflecting on their own methods

C. ***Consistency Matters.***

When I began using this strategy, I held number talks between two and three times per week. After a few months, I increased them to four and sometimes five days. When number talks are used frequently and consistently, more students participate, and the quality of the conversation increases.

D. ***Explain Less, Listen More.***

As much as you can, avoid explaining. This activity works best the more student-centered it is. Students will usually come up with their own strategies after a while. If tempted to explain a process, the teacher should think of a way to get students to do it instead. I asked students to explain each other's work. I ask them what works and what does not. I ask them how they know if something makes sense. I ask them which strategies require the fewest steps, and which are easy to do mentally. These types of questions usually helped spark new ideas.

E. ***Wait, Wait, and Wait Some More*.**

Give students as much time as they need to solve the problem. It is important to reinforce that number talks are not about being fast. Use wait time when students are explaining their work. If the wait time gets too long, this usually indicates that the problem is not right for this group on this day.

F. ***Choose the Right Problems.***

Number talks are an excellent formative assessment tool, but not all arithmetic problems will generate a great discussion. Choose problems that lend themselves to multiple strategies and be aware of the level of challenge. If students are not yet ready to solve *15 x 160*, use a simpler problem at first. Once they are comfortable with a few strategies, they will be able to transfer that to more challenging problems.

G. **My *Final Thoughts About Number Talks***

Using number talks in my classroom allowed my students to be creative when solving arithmetic problems. Despite incorporating number talks into my daily class routine, not all students fully participated. Some showed little growth in reasoning or conceptual understanding. The students who participated frequently learned multiple ways to solve problems. They compared different methods and explored efficient strategies. They encouraged each other to provide clear explanations and explained their own thinking. These students showed growth in their understanding of arithmetic and became more flexible in problem solving.

Number talks helped me practice effective questioning strategies and encouraged the students to take risks. Because these talks were strictly about arithmetic, we could work on a concept for as long as it took students to understand it. Frequent number talks helped the class practice discussion skills and grow as a learning community. I enjoyed seeing my students explore and explain creative solutions to problems. Many students developed a sense of ownership in the classroom as they found their own way to look at numbers. Through this process, many of my students became mathematically powerful.

VII. Policy Recommendations

A. Classrooms Teachers:

- Encourage regular discussion of math concepts with your students.
- Invite more flexibility in problem solving.
- Engage students in learning that helps them make sense of problems.
- Incorporate new strategies into everyday teaching activities.

B. Schools:

- Prioritize student discussions in math classrooms.
- Provide professional development on how to effectively use discussion techniques in math classes.
- Provide opportunities for teachers to observe and discuss strategies.
- Ensure that teachers at all levels are knowledgeable about number sense.

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